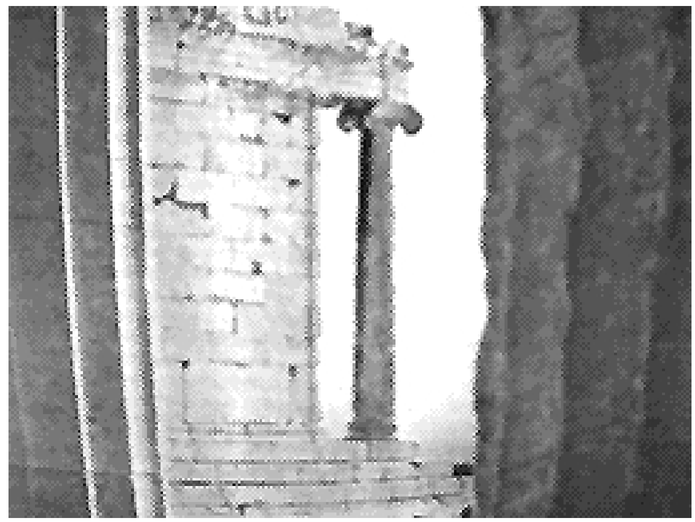
Structures I Lectures

Lecture 1:

Four on the Floor! (Strength vs. Stability)



There are two issues that will be emphasized throughout this course that are crucial to the understanding of Architectonics:

- **Strength**
  the capacity of the individual elements, which together make up a structural system, to withstand the load that is applied to it.

- **Stability**
  the capability of a structural system to transmit various loadings safely to the ground.

These two critical issues are experienced daily from the moment that an individual is born. A newborn baby cannot even hold its own head upright. The large mass of the head requires a support system that has sufficient strength to enable the head to maintain its stability. This steadily increases as the bones, muscles and tendons of the skeletal and muscular systems increase in strength. Eventually the extra support provided by the arm or hand is no longer needed. The first challenge posed by gravity is overcome.

Crawling on four points of support proves to be a very stabile situation for quite a long time. The "leap" to the unstable two point stance is the next development in our understanding of the influence of gravity. Again, the structural system must develop to the point that the individual elements of the system have acquired sufficient strength. The first steps are made: an action of supreme coordination of hundreds of elements that becomes second nature to homo sapiens.

The list can be extrapolated to touch on many aspects of the human experience; riding tricycles and bicycles, jumping on trampolines, exercising on parallel bars, sliding on ice skates, sailing in a heavy wind, rocking a small boat, ... the list is endless. These are part of the human experience and each and every one rely on an inherent understanding of strength and stability.

How many times has a parent scolded a child to "put four on the floor!!!"? What the parent really means to say is, "if you do not put all of the legs of your chair on the ground, you are going to tip over!" Both strength and stability issues are addressed in this simple exclamation. Under normal conditions, the elements which make up the chair (its legs, bracing and seat) can easily resist the implied vertical loads. The **strength** of the individual elements of the chair have been designed for this type of static load. The seat (as a horizontal load-bearing element) must transfer its load through a connection to the legs (vertical load-bearing elements). Granted, some chairs will withstand a greater load than others, but they all resist the pull of gravity on the person sitting in them. If the legs cannot support the applied load they will fracture or break. These are examples of strength failure.

The **stability** of the system of elements depends upon the orientation of the chair in space. When it stands upright, on all four legs, it is a stable stystem. If it is on it's side, the chair might not be able to resist the loads for which it was designed. As it is tilted onto the back two legs, the structural system loses its equilibrium. At a certain point the chair as a system becomes unstable, fails and gravity pulls the supported load to the ground. This is a stability failure. In this type of failure, the individual elements retain their strength even as the system fails. The chair (system) could also have failed if the two supporting legs had experienced a strength failure (broken).

In each of these situations the chair, as a structural system, has reached the limit of its strength. As the saying goes, a chain (structural system) is only as strong as the weakest link (element)!

Any structural system can be studied in light of these two issues. For example, the column of the Greek temple shown above is an element that can experience a strength (crushing) failure, or a system (buckling) failure. It is/was part of a larger structural system.

Questions for Thought

What are some structural systems that you can see around you as you sit? How could they fail? How would one of Marcel Breuer's stainless steel tube chairs be discussed in relation to the issues of strength and stability? How would you describe the working of the support systems of your body in relation to the issues of strength and stability? How would you describe the basketball backboard and supporting structure shown in terms of strength and stability?

Additional Reading

Lecture 2:

What is STRUCTURE?

One of the greatest problems of designing today is the fact that engineers can solve ANY problem. Anything can be built. Structural "realities" are perceived as no longer imposing limitations upon the design architect. Form does not have to be dictated by structure or even follow a function. Many of the seemingly undeniable "truths" of architectural design have been rendered meaningless. Yet, gravity persists despite this incredible freedom of choice. Buildings must stand up at the end of a real or virtual working day.

Architectural design cannot be based soley upon one of the many aspects that make up the profession. It surely should never be based on architectonics alone. Yet, structure is the very raw material of building. To use structure without understanding its implications is irresponsible and results in meaningless formalism. An architect is supposed to be a specialist in building, not just a creator of arbitrary form. The word **structure** can be used alone or in conjunction with many other descriptive words. Dictionaries can be consulted to find the following definitions:

manner of construction
the arrangement of particles or parts in a substance or body
arrangement or interrelation of parts as dominated by the general character of the whole
the aggregate of elements of an entity in their relationships to each other

the composition of conscious experience with its elements and their combinations
something that is constructed
something that is arranged in a definate pattern of organization
the action of building

There are multitudes of different scales at which one should perceive structures. Each scale reveals beauty and provides an amazing amount of information at the same time. Seeing the information at each level of perception is critical. Learning to see the structure of the world around us is an important part of life and of this course. It is critical to the success of an architect that she/he be able to see beyond the skin of a building; beyond the surfaces of a space and into the load-bearing structure. This is the fabric from which space is molded. Understanding the nature of the fabric enables one to create the seams between spaces. Understanding the load-bearing structure of a building is to understand the space that is being created.

There is a fundamental rightness in a structurally correct concept. It leads to an economy of means that can be understood by all. Designs which are inherently structurally correct are often perceived as objects of great beauty, even if only truely comprehended by few. One can find structure in everything. Look at landscapes, cities, roofs, walls, and at the veins in a leaf from both afar and as close as you can. Record what you see. What are the similarities? What is unique about each? Look at the:

- external expression of internal structure
- relationship between natural and built forms
- relationship between size and internal forces
- articulation and supporting structure of vertical surfaces
- articulation and supporting structure of horizontal surfaces

- nature of scale in relation to the elements of a system
- nature of scale in relation to a system
- openings in a wall
- relationship between loading and structural form

This is an image of a stucture found in an empty lot in Hong Kong in 1996. Examine it in light of the the list of thoughts above. The wood truss clearly articulates the relative magnitude of the forces. There are many structures to be seen in this image. Here is a larger JPEG if you want to look at it in more detail.

Questions for Thought

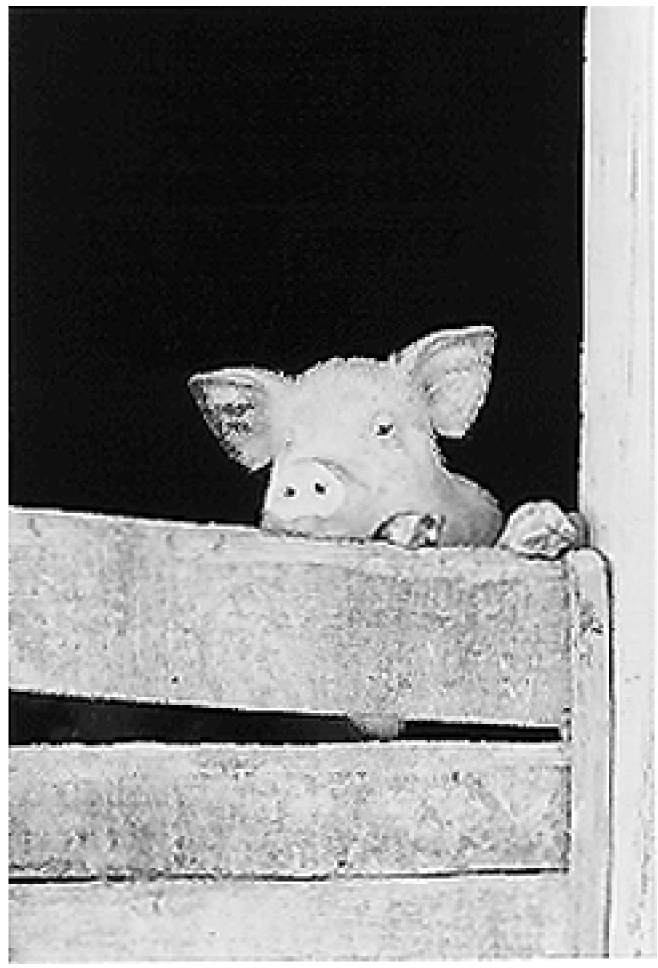
What other structures do you see? What would be the most logical materials to build these different structures? How would that change if the site was located in Borneo? or in Sweden? or in New York? Can you find a similar type of situation around you?

Additional Reading

Mainstone, Rowland. *Developments in Structural Form.* 1975. Chapter 1

Lecture 3:

The Role of Structural Failure



The concepts of element strength and system stability are learned at an early age. They often appear in children's stories that have been repeated for decades. One favorite in North America is the story of the "Three Little Pigs." Each pig desired to build a house to protect itself from the Big, Bad Wolf. One pig built a house of straw while another built a house of sticks and twigs. The third pig erected a house of brick. The story follows that the big, bad wolf came to the homes of the three pigs. He drew a huge breath and huffed and puffed until he blew the first house down. He chased the pig into the second house where he repeated his imitation of a typhoon. This house was also blown over and the pigs fled into the brick house. In front of the brick house the wolf hyperventilated. His grandest attempts to blow over this ediface failed and so he did not have pork for dinner. The story teaches children that a brick building is the strongest and most stable becuase it is the only one which could resist the huffing and puffing. The straw and stick buildings were blown away in the wind and only within the brick structure do the three pigs find their security.

This lesson is correct if the load that is of greatest concern is a tornado, typhoon, hurricane or similar lateral load. However, the brick house would have been the worst structure to live in if it was built in an earthquake zone. In fact, an unreinforced masonry building is much more life threatening in an earthquake when compared to a straw hut. Then again, another response would be appropriate if the house was in a region where winds were not a real problem, but extreme temperature variations were critical. The same would be true if mobility was the most significant issue. Each of these are specific loading conditions that could lead to structural failure. It is critical that the appropriate loads and loading combinations are considered for each and every structure depending on use and location. Every one is different!

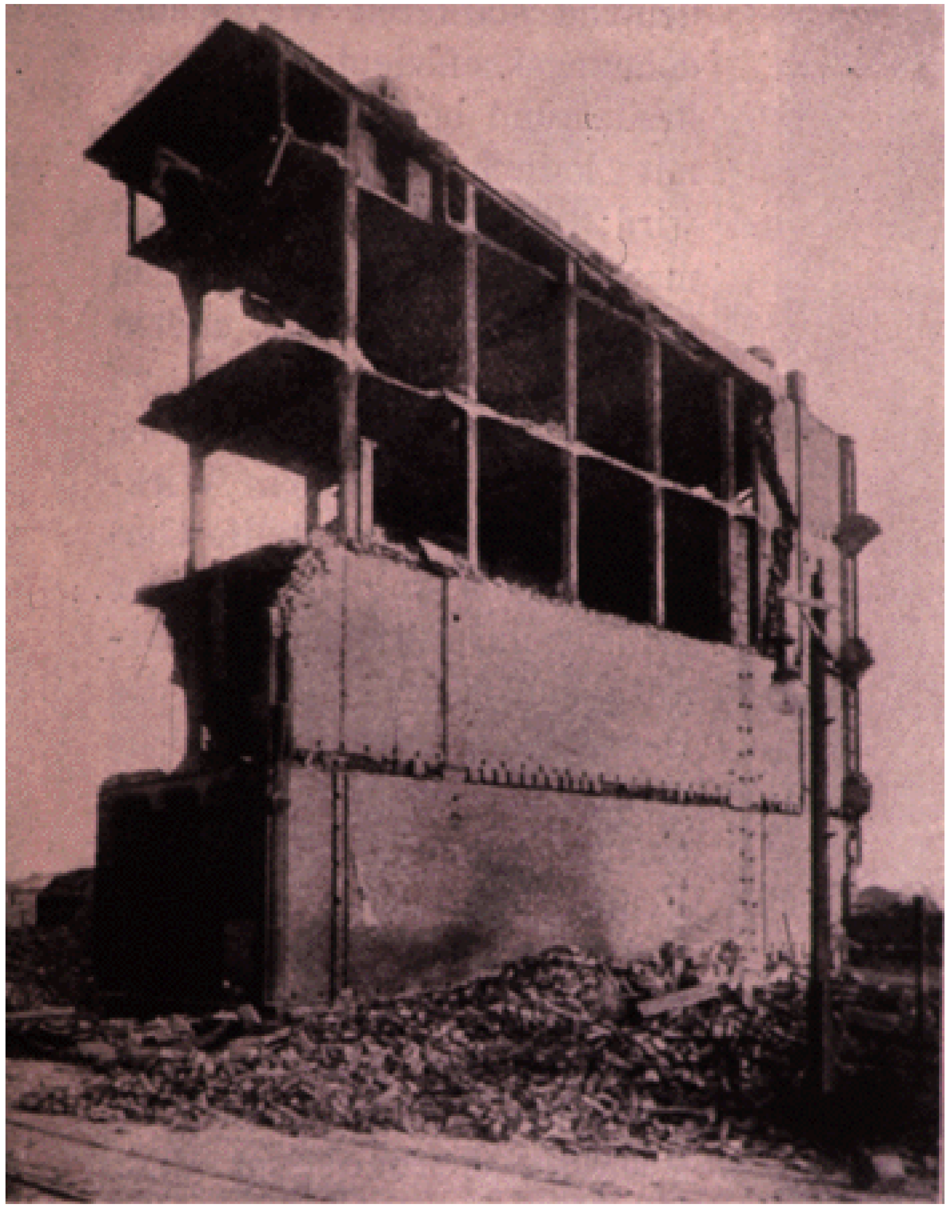
"Humpy Dumpty" is another childrens' tale with a structural lesson. He was a good egg who sat on a wall for quite a while. One day he lost his balance and fell to the ground where all of the king's men could not put Humpty back together again. The egg shell is a wonderfully efficient, naturally occuring thin structure that is extremly strong under evenly distributed loading conditions. One type of load which a thin shell cannot resist is a sharp point load. Thus, when Humpty fell to the ground he loaded his wonderful structure in a way that caused a strength and system failure.

Building Codes are often considered to be the bane of architectural designers. However, they have been a part of the profession for thousands of years. The illustration above is a part of the Code of Hammurabi that was written about 2200BC. The Code can be translated as:

> If a builder builds a house for a man and does not make its construction firm and the house collapses and causes the death of the owner of the house - that builder shall be put to death.
>
> If it causes the death of a son of the owner - a son of that builder shall be put to death .
>
> If it causes the death of a slave of the owner - the builder shall give the owner a slave of equal value.
>
> If it destroys property - the builder shall restore whatever it destroyed and because the builder did not make the house firm, shall rebuild the house which collapsed at his own expense.
>
> If a builder builds a house and does not make its construction meet the requirements and a wall falls in - that builder shall strengthen the wall at his own expense.

The negative incentive of the Code of Hammurabi does not bode well for innovation. However, it clearly demonstrates the point that building codes have existed for a long time, and are not likely to go away. Structural designers are often considered to be a very conservative group. One of the reasons for this could be found in such a building code. In addition, when a building experiences structural failure the results are usually quite different than if the air conditioning system fails. Although some aspects of the building codes seem ridiculous, they are in place to guarantee the public's safety and should be respected.

Each and every one of us is a structural designer in their own way. The act of walking down a set of stairs or pushing a wheelchair forward or swimming across a pool requires an understanding of gravity, the flow of forces and how to work with them.



Petroski notes: "The Concept of Failure is central to understanding engineering, it has as its first and foremost objective the obviation of failure. Design, even structural design, is a human endeavor and thus it is subject to error. Due to this, some designs are destined to fail. This can lead to a loss of life which in itself is tragic, but a deeper unforgivable tragedy exists when the lessons of the failure are understood and allowed to occur again."

Questions for Thought

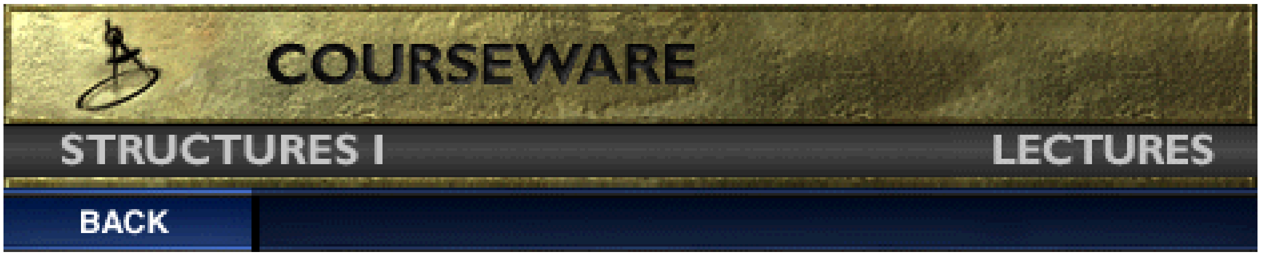
What part does, or did, "trial and error" play in the development of the understanding of structural behavior? How many Cathedrals collapsed before the master builders determined the correct arch for the ceilings of the Gothic Cathedrals? How many times did it take for you to learn how to do a specific physical activity that required extreme concentration and balance? A building only has one chance. Can you remember the last time that you read about a building failure? What failed? Why did that part of the building fail? Was the failure due to limitations of human understanding? or was it due to human error or neglegence? or perhaps due to greed? Was the failure due to an unexpected load? What kind of unexptected load created the failure of the buildings around the frame in the previous picture?

Failure can also be illustrated with the following exercise. Take ten small paper clips. Twist each one back and forth and count the number of turns that it takes for each paper clip to fail. Note the variance in what seems to be exactly the same material for each one. What role does statistics play in structural design?

Additional Reading

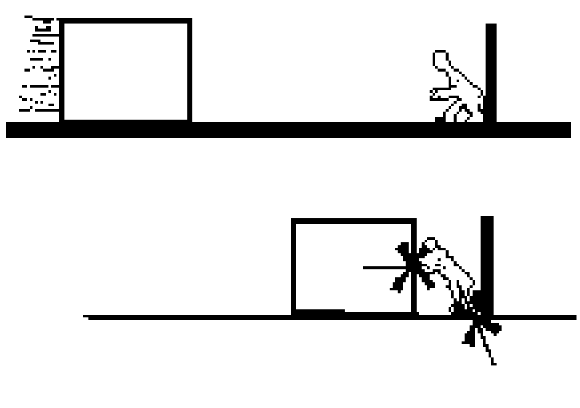
Petroski, Henry. *To Err Is Human; The Role of Failure in Design.*

EXTERNAL LINK The Quebec Bridge Failure

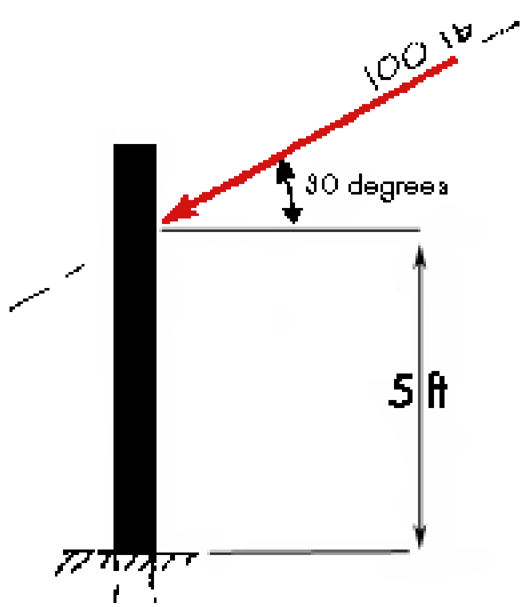


Lecture 4:

What is a Force?



The one constant around the world is the action of gravity upon each and every structure that is erected. The primary function of all structural design is to make a building stand-up. Understanding architectonics will enable a designer to include these issues as part of a design language that will create a significantly clearer architectural expression. Primary to this study is the concept of a **force**. A force is actually a very abstract conception. It can be defined, but it cannot become physically apparent until it meets resistance. Imagine a six foot tall block of ice sliding along a frictionless surface laid inside of a hockey rink. If a person props herself against the wall and tries to stop the ice she will then perceive the force imparted by the block of ice. She transfers the force of the block of ice into a force that moves into the ground. Thus, if the ground could actually experience a push, it would as well as the slide is arrested.



A "force" is an action that changes, or tends to change, the state of motion of the body upon which it acts. It is a vector quantity that can can be represented either mathematically or graphically.

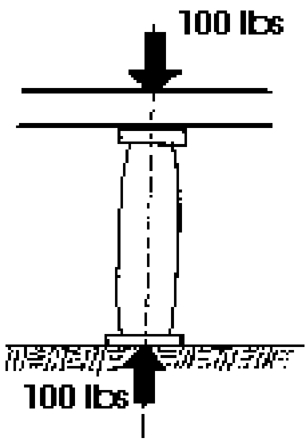
A complete description of a force MUST include its:

1. MAGNITUDE
2. DIRECTION and SENSE
3. POINT OF APPLICATION

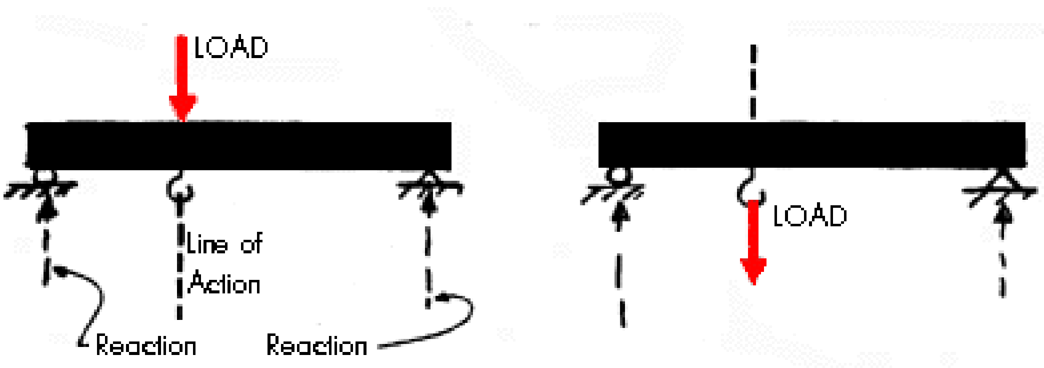
The Magnitude is most often expressed in the units of pounds (lbs), newtons(N), kilo-newtons(KN) or kilo-pounds(KIPS; 1 Kip = 1000 lbs). The magnitude is represented graphically by the scaled length of the arrow which represents the force. Graphic statics depends upon the accurate representation of the magnitude of each force acting upon a body.

The Direction of a force is discerned by oberserving the line of action of the vector of the force. The Sense refers to the direction of the movement of the vector along that line of action. The sense is always represented on the vector by an arrowhead. These two attributes can be either a written verbal description or more conveniently expressed in terms of 360 degrees. In the later case, one begins with zero and increases clock-wise with the direction of the arrowhead until 360 is reached. The sense of the force can also be expressed as a positive or negative sign. This is especially useful when combining forces algebraically. It is very important not to confuse the direction and sense. The direction always relates to the line of action of the vector, and the sense is the way in which the vector would move along that line. In the example, the sense of the 100 lb force would be "down and to the left" or "210 degrees."

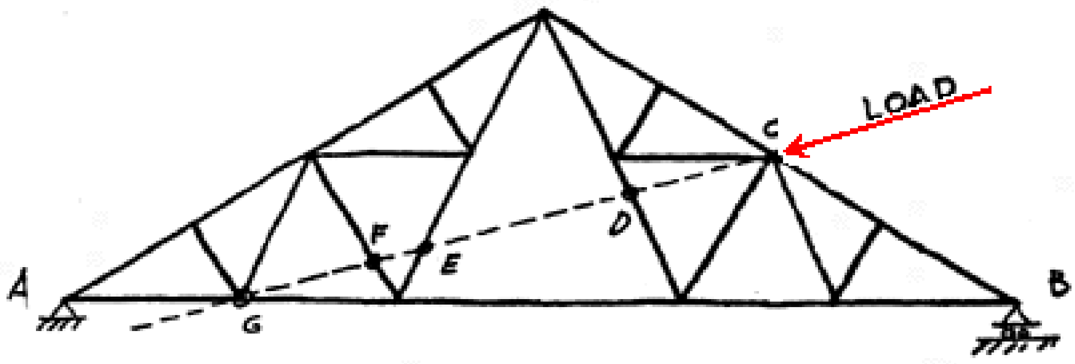
The Point of Application is often overlooked in the description of a force. However, it is fully as important as the magnitude of the force! It is the exact location of the application of a force on a body. It can either be a relative measurement or a set of coordinates.



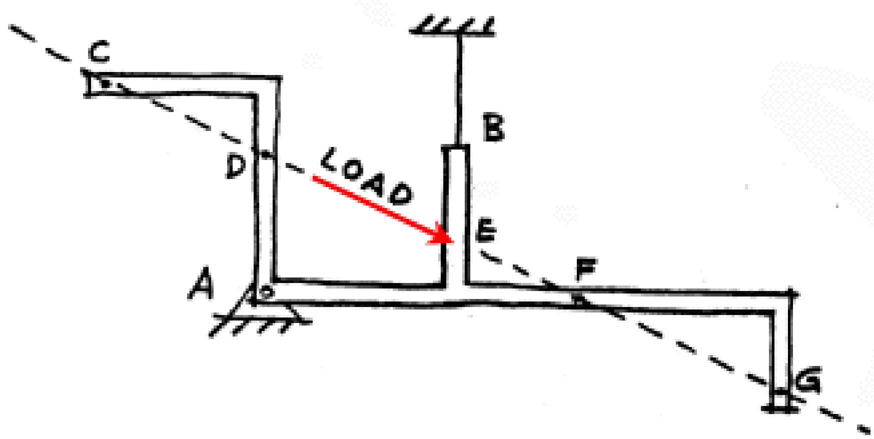
A 100 kip force is applied to the stone column in the diagram. The colum will experience this force at every point along the line of action of the force. As a matter of fact, the force will also be transferred to the ground which is supporting the column. Thus, the earth below the column along the line of action of the force will also experience the 100 kip load. This illustrates the **Principle of Transmissibility**. The point of application of an external force acting on a body (structure) may be transmitted anywhere along the force's line of action without affecting the other external forces (reactions and loads) acting on that body. This means that there is NO NET CHANGE in the static effect upon any body if the body is in equilibrium. This can be illustrated with the following diagram.



Assume that a beam is supported at its ends. A load is applied to the top of the beam that is acting downward. This load could be a person standing on the beam. The load creates reactions that push up at the two points of support. The line of action of the load (person) on the beam also passes through a hook that is attached to the underside of the beam. Now, if the person standing on top of the beam would climb down and hold on to the hook exactly below the point where they were previously located so that the lines of action were exactly the same, the reactions at the ends of the beams would not change. This is because the load of the person is still acting along the same line of action. As long as a load is applied at any point along the line of action the external reactions will not change.



The truss above is loaded with a force that is applied at point C. This load creates reactions at the two supports A and B. The load on the truss could move anywhere along the line of action and the external reactions at A and B would remain the same. That means that if the load was applied at points D, E, F or G the reactions at A and B would not change. Note that the only point of discussion at this moment is the fact that the external reactions will not change. It is clear that the internal forces will vary greatly within the truss as the force is moved along the line of action. This issue will be discussed in a later lecture.



This strangly shaped beam is another example of a structure that is loaded at a specific point, namely E. In order for the structure to remain at rest there must be reactions of some kind at points A and B. The reaction at B is a tension reaction since it is a cable. Again, if the load was applied at points C, D, E, F or G on this rigid body (structure) the reactions at A and at B would remain exactly the same.

ONLY the EXTERNAL forces (reactions) remain unchanged. Some of the internal resisting forces within the elements of the structure change as the load is applied at different points along its line of action. This illustrates one important difference between INTERNAL and EXTERNAL forces.

The Principle of Transmissibility applies to any body (blobs, balloons, simple beams, crooked beams, trusses, shells, etc.). It is independent of the body's size or shape.

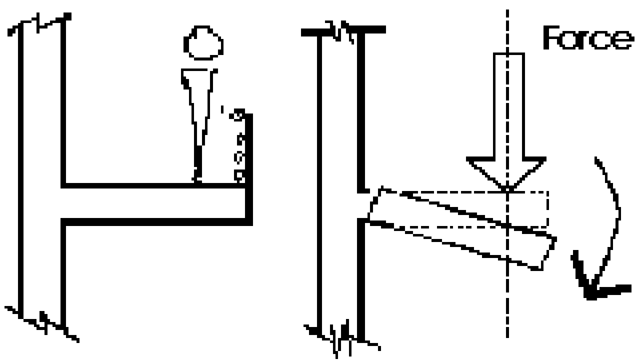
Questions for Thought

What happens to the reactions at A and B if the sense of the applied load would be reversed in both of preceeding diagrams?

Additional Reading

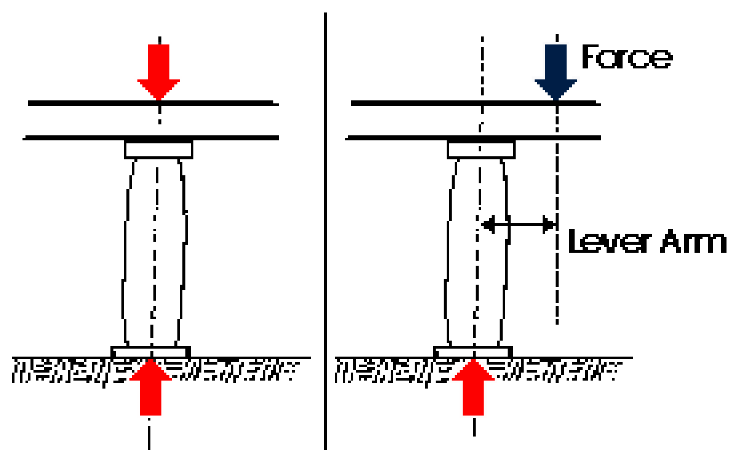
Lecture 5:

Moments



The **Moment** of a force is a measure of its tendency to cause a body to rotate about a specific point or axis. This is different from the tendency for a body to move, or translate, in the direction of the force. In order for a moment to develop, the force must act upon the body in such a manner that the body would begin to twist. This occurs every time a force is applied so that it does not pass through the centroid of the body. A moment is due to a force not having an equal and opposite force directly along it's line of action.

Imagine two people pushing on a door at the doorknob from opposite sides. If both of them are pushing with an equal force then there is a state of equilibrium. If one of them would suddenly jump back from the door, the push of the other person would no longer have any opposition and the door would swing away. The person who was still pushing on the door created a moment.

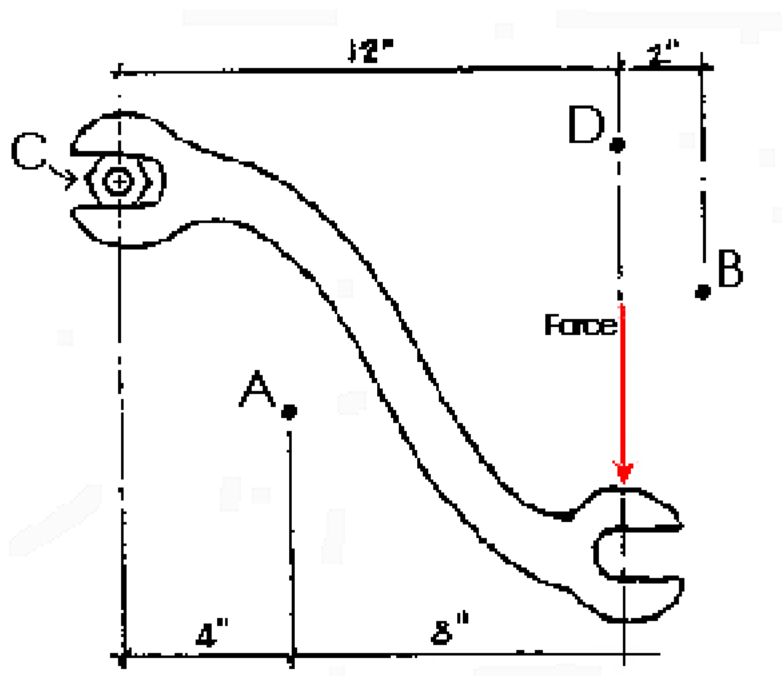


The magnitude of the moment of a force acting about a point or axis is directly proportinoal to the distance of the force from the point or axis. It is defined as the product of the force (F) and the moment arm (d). The **moment arm** or **lever arm** is the perpendicular distance between the line of action of the force and the center of moments. The **Center of Moments** may be the actual point about which the force causes rotation. It may also be a reference point or axis about which the force may be considered as causing rotation. It does not matter as long as a specific point is always taken as the reference point. The latter case is much more common situation in structural design problems. A moment is expressed in units of foot-pounds, kip-feet, newton-meters, or kilonewton-meters. A moment also has a sense; it is either clockwise or counter-clockwise. The most common way to express a moment is

Moment = Force x Distance

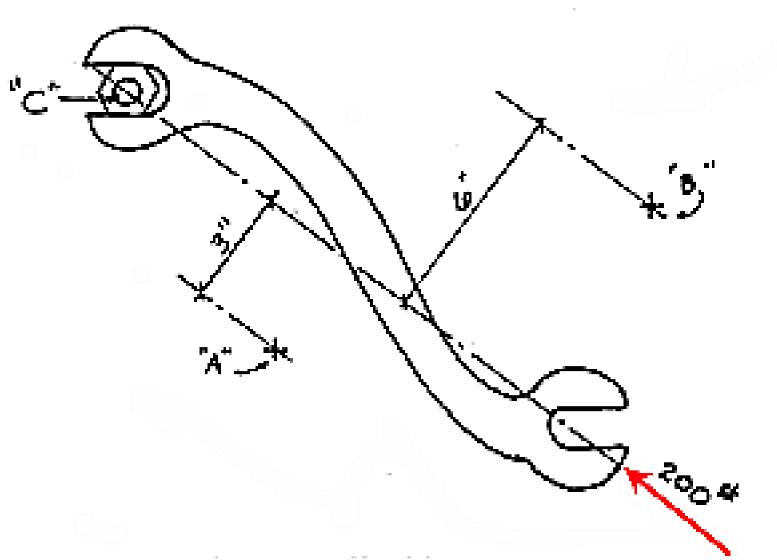
M = F x d

The example shown is a wrench on a nut (at C) that has a force applied to it. The force is applied at a distance of 12 inches from the nut. The center of moments could be point C or points A or B. The moment arm for calculating the moment around point C is 12 inches. The magnitude of the moment of the 100 pound force about point C is 12 inches multiplied by the force of 100 pounds to give a moment of 1200 inch-pounds (or 100 foot-pounds). Similarly, the moment about point A can be found to be 800 inch-pounds.

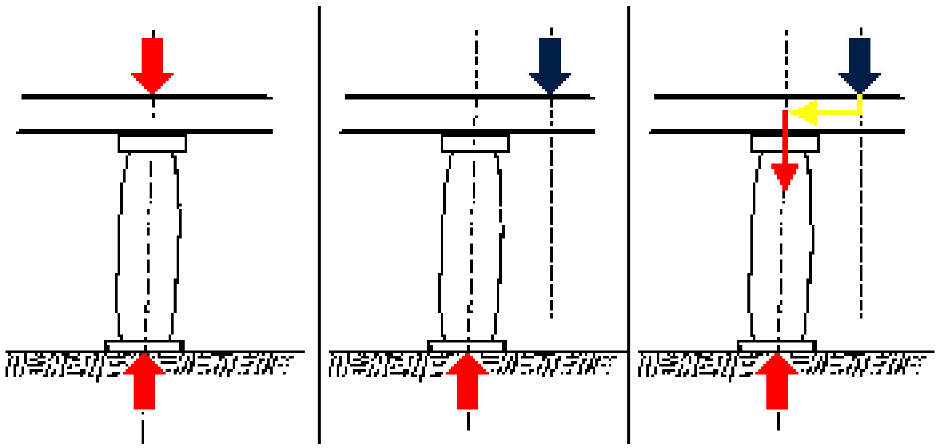


The direction of the rotation is important to understand in order to describe its effect on the body (structure). A moment will cause either a clockwise or counter-clockwise rotation about the center of moments. It is essential that the direction of rotation about the center of moments be understood. Since an international convention does not exist, in this course, a clockwise rotation about the center of moments will be considered as positive; a counter-clockwise rotation about the center of moments will be considered as negative.

A moment causes a rotation about a point or axis. Thus, the moment of a force taken about any point that lies on its own line of action is zero. Such a force cannot cause a rotation because the moment arm is non-existant.



In this second example, a 200 pound force is applied to the wrench. The moment of the 200 pound force applied at C is zero because 200 pounds x 0 inches = 0 inch-pounds (Fxd = M). In other words, there is no tendency for the 200 pound force to cause the wrench to rotate the nut. One could increase the magnitude of the force until the bolt finally broke off.



A moment can also be considered to be the result of forces detouring from a direct line drawn between the point of loading of a system and its supports. In this case, the blue force is an eccentric force. In order for it to reach the base of the column, it must make a detour through the beam. The greater the detour, the greater the moment. The most efficient structural systems have the least amount of detours possible. This will be discussed in more detail in Lecture 37 and later courses.

There are cases in which it is easier to calculate the moments of the componenets of a force around a certain point than it is to calculate the moment of the force itself. It could be that the determination of the perpendicular distance of the force is more difficult than determining the perpendicular distance of components of the force. The moment of several forces about a point is simply the algebraic sum of their component moments about the same point. When adding the moments of componenets, one must take great care to be consistant with the sense of each moment. It is often prudent to note the sense next to the moment when undertaking such problems.

EXAMPLE PROBLEM Combined Moments

EXAMPLE PROBLEM Moments on a Beam

Any difficulty with calculating a moment can usually be traced to one of the following:

1. The center of moments has not been correctly established or clearly understood.
2. The assumed moment arm is not the PERPENDICULAR distance between the line of action of

the force and the center of moments.
3. The direction, or sense, of the rotation has been ignored or misunderstood.

Questions for Thought

What is the moment about point B and about point D for both of the cases shown in the wrench example above? How could adding an extension to the end of the wrench help turn a rusted bolt? What kind of structural systems would have the least number of "detours?"

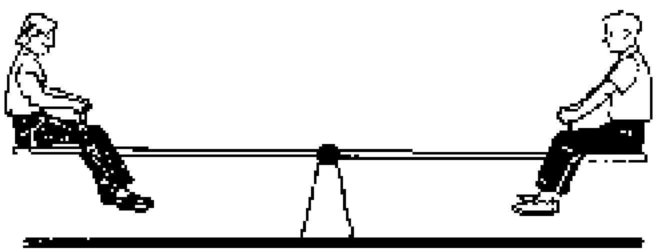
Additional Reading

Shaeffer, R.E. *Elementary Structures for Architects and Builders*. pp. 33-39.

Lecture 6:

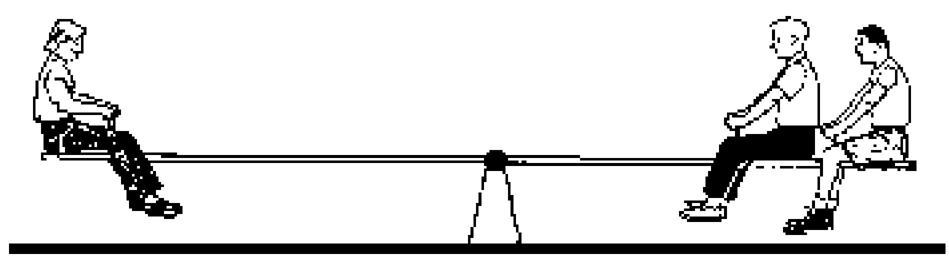
Equilibrium

Architectural structures are normally stationary. Most clients, building officials and designers prefer that their structures remain static rather than move dynamically. There are specific loading conditions which are dynamic loads, but in each and every case a return to a stable and static state is desireable. Such a condition is known as equilibrium.

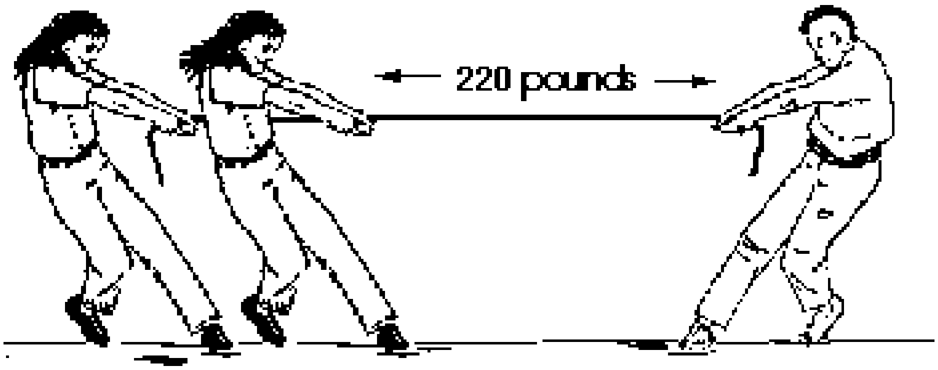


Various states of static equilibrium are experinced throughout one's life. Think of the "teeter-totter" at a playground or of a game of "tug-of-war." In the first case, two or more individuals sit upon a board which has been fixed to a fulcrum which allows rotation. If each of the individuals on the teeter totter weight exaclty the same amount and sit at exactly the same distance from the fulcrum the teeter-totter will not move. A state of equilibrium has been achieved. The two will remain at rest until an action takes them out of equilibrium.

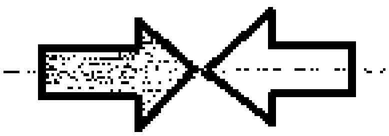
Such an action could be the addition of another person to the system or it could be that one of the original two would change their position slightly. In either case, the teeter-totter would most likely swing to one side and rest upon the ground. A new state of equilibrium would have been found.



In this case, another boy climbed on behind the one already sitting on the right. In order to put the system back into equilibrium the girl on the left had to move backwards along the board until she was far away. The moment two boys created around the fulcrum doubled when the second boy climbed on. The girl knew that the only way that she could increase the magnitude of the moment she created would be to increase the moment arm. Thus she moved back until she was twice as far from the fulcrum. Now the system would be back in equilibrium.



Another example of a state of equilibrium is the game of "tug-of-war." A rope is pulled taught between two teams; each pulls with a force that equals the force of the oppostie team. Assume in the figure that each team is pulling with a force of 220 pounds. As long as each team maintained a pull of 220 pounds the system is in equilibrium. If during this time a device would be inserted between the two teams to measure the magnitude of the tension force that the rope has anywhere along its length, it would read 220 pounds at each and every point. This would be true at ANY point along the rope.



A structure is in **equilibrium** when all forces or moments acting upon it are balanced. This means that each and every force acting upon a body, or part of the body, is resisted by either another equal and opposite force or set of forces whose net result is zero. Issac Newton addressed this issue when he noted that a body is at rest will remain at rest until acted upon by an external force. Every structure that can be seen to remain standing on a daily basis is in equilibrium; it is at rest and each of its members, combination of its members or any part of a member that is supporting a load are also at rest. There is a net result of zero in all directions for all of the applied loads and reactions.

In both of the the illustrations above, the split bamboo beam is held in a state of equilibrium. The beam of the top figure illustrates the sytsm at rest under it's own weight. The lower figure shown another state in which a pair of scissors sits at one end and a thermos of coffee on the opposite side near the fulcrum. All of the forces and moments are balanced so that the system is in a stable equilibrium.

There are two types of equilibrium; External and Internal. External equilibrium encompasses the loads upon, and reactions of, a structural system as a whole. Internal equilibrium describes the various forces the are acting within every member of the system. There are conditions of equilibrium that must be satisfied for each case. These are:

Sum of All Vertical Forces (F_y) = 0

Sum of All Horizontal Forces (F_x) = 0

Sum of All Moments (M_z) = 0

(Sum of All Forces (F_z) = 0)

(Sum of All Moments (M_y) = 0)

(Sum of All Moments (M_x) = 0)

These six equations are all that can be used to determine every one of the forces that are acting with a structure. They are few, but very powerful. The first three are the most common equations and will be utilized in all of the problems asociated with thid course. The other three are only necessary when considering three-dimensional force systems.

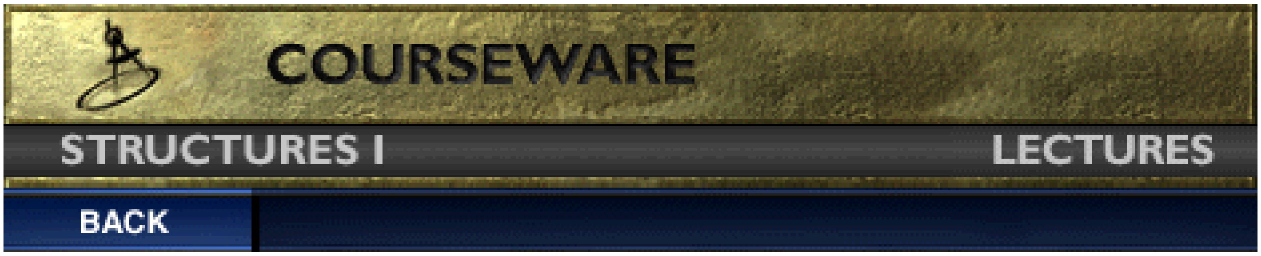
EXAMPLE PROBLEM Force System Equilibrium

EXAMPLE PROBLEM Teeter Totter Equilibrium

Questions for Thought

What other games developed your sense of equilibrium during play as a child? How is equilibrium maintained by a large Oak tree when it must resist the onslaught of a strong wind? How is equilibrium maintained by a tall Douglas Fir tree when it must resist the onslaught of a strong wind? How is equilibrium maintained by a building when it must resist the onslaught of a hurricane? How is equilibrium maintained when a person rides a bicycle?

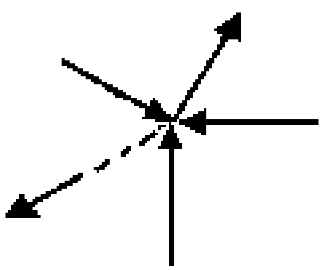
Additional Reading



Lecture 7:

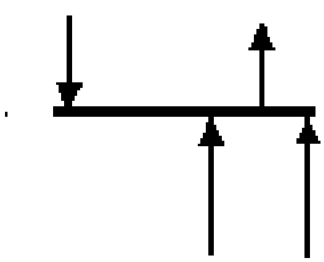
The Elements of Coplanar Force Resolution

There are many ways in which forces can be manipulated. It is often easier to work with a large, complicated system of forces by reducing it an ever decreasing number of smaller problems. This is called the "resolution" of forces or force systems. This is one way to simplify what may otherwise seem to be an impossible system of forces acting on a body. Certain systems of forces are easier to resolve than others. **Coplanar** force systems have all the forces acting in in one plane. They may be concurrent, parallel, non-concurrent or non-parallel. All of these systems can be resolved by using graphic statics or algebra.



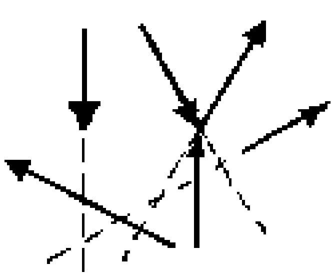
Concurrent

A **concurrent** coplanar force system is a system of two or more forces whose lines of action ALL intersect at a common point. However, all of the individual vectors might not acutally be in contact with the common point. These are the most simple force systems to resolve with any one of many graphical or algebraic options.



Parallel

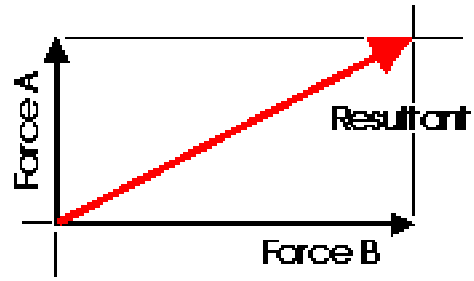
A **parallel** coplanar force system consists of two or more forces whose lines of action are ALL parallel. This is commonly the situation when simple beams are analyzed under gravity loads. These can be solved graphically, but are combined most easily using algebraic methods.



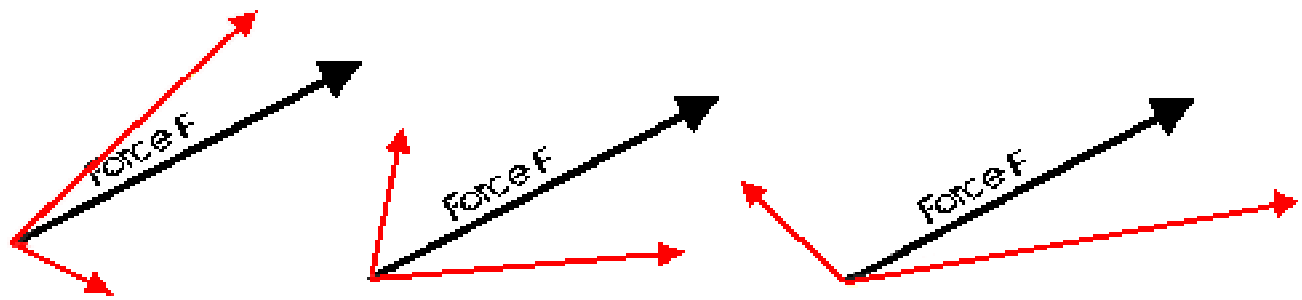
Non-Concurrent

The last illustration is of a **non-concurrent and non-parallel** system. This consists of a number of vectors that do not meet at a single point and none of them are parallel. These systems are essentially a jumble of forces and take considerable care to resolve.

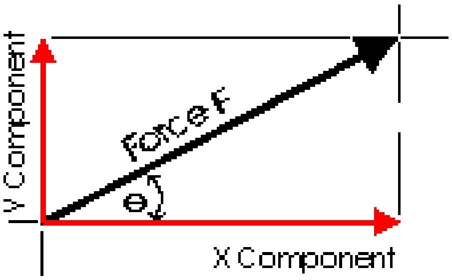
Non-Concurrent Non-Parallel



Almost any system of known forces can be resolved into a single force called a resultant force or simply a Resultant. The **resultant** is a representative force which has the same effect on the body as the group of forces it replaces. (A couple is an exception to this) It, as one single force, can represent any number of forces and is very useful when resolving multiple groups of forces. One can progressively resolve pairs or small groups of forces into resultants. Then another resultant of the resultants can be found and so on until all of the fources have been combined into one force. This is one way to save time with the tedious "bookkeeping" involved with a large number of individual forces. Resultants can be determined both graphically and algebraically. The graphical methods that will be discussed in Lecture 8 are the Parallelogram Method and the Triangle Method. It is important to note that for any given system of forces, there is only one resultant.

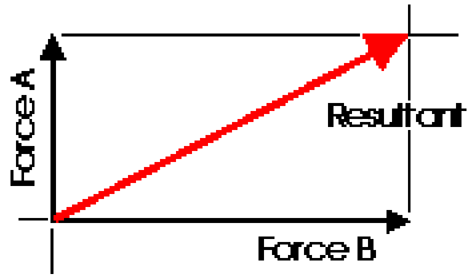


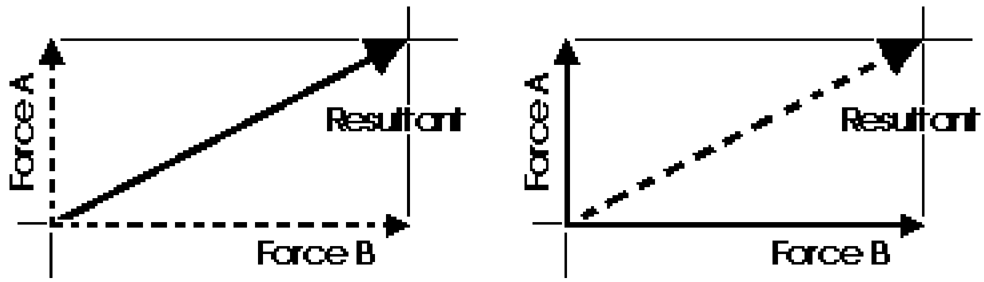
It is often convenient to decompose a single force into two distinct forces. These forces, when acting together, have the same external effect on a body as the original force. They are known as **components**. Finding the components of a force can be viewed as the converse of finding a resultant. There are an infinant number of components to any single force. And, the correct choice of the pair to represent a force depends upon the most convenient geometry. For simplicity, the most convenietnt is often the coordinate axis of a structure.



This diagram illustrates a pair of components that correspond with the X and Y axis. These are known as the **rectangular componants** of a force. Rectangular components can be thought of as the two sides of a right angle which are at ninety degrees to each other. The resultant of these components is the hypotenuse of the triangle. The rectangular components for any force can be found with trigonometrical relationships: $F_x = F \cos \theta$, $F_y = F \sin \theta$.

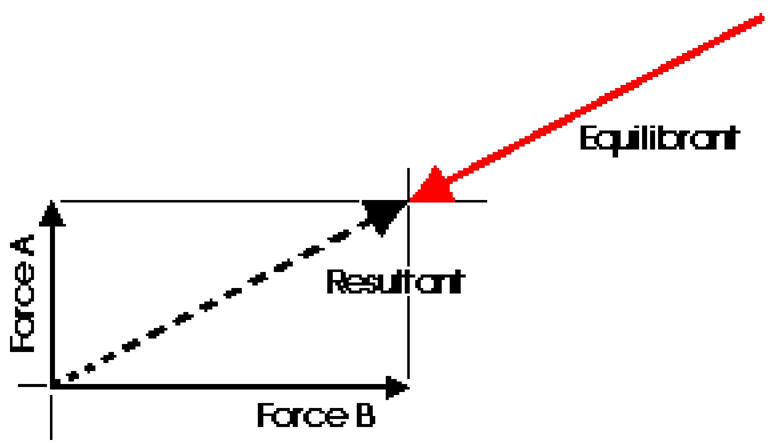
There are a few geometric relationships that seem to common in general building pratice in North America. These relationships relate to roof pitches, stair pitches, and common slopes or relationships between truss members. Some of these are triangles with sides of ratios of 3-4-5, 1-2-sqrt3, 1-1-sqrt2, 5-12-13 or 8-15-17. Commiting the first three to memory will simplify the determination of vector magnitudes when resolving more difficult problems.





When forces are being represented as vectors, it is important to should show a clear distinction between a resultant and its components. The resultant could be shown with color or as a dashed line and the components as solid lines, or vice versa. NEVER represent the resultant in the same graphic way as its components.

Any concurrent set of forces, not in equilibrium, can be put into a state of equilibrium by a single force. This force is called the **Equilibrant**. It is equal in magnitude, opposite in sense and co-linear with the resultant. When this force is added to the force system, the sum of all of the forces is equal to zero. A non-concurrent or a parallel force system can actually be in equilibrium with respect to all of the forces, but not be in equilibrium with respect to moments.



Graphic Statics and graphical methods of force resolution were developed before the turn of the century by Karl Culmann. They were the only methods of structural analysis for manny years. These methods can help to develop an intuitive understanding of the action of the forces. Today, the Algebraic Method is considered to be more applicable to structural design. Despite this, graphical methods are a very easy way to get a quick answer for a structural design problem and can aid in the determination of structural form.

EXAMPLE PROBLEM Resultants of Forces

EXAMPLE PROBLEM Components of Forces

EXAMPLE PROBLEM Equilibrants of Forces

Questions for Thought

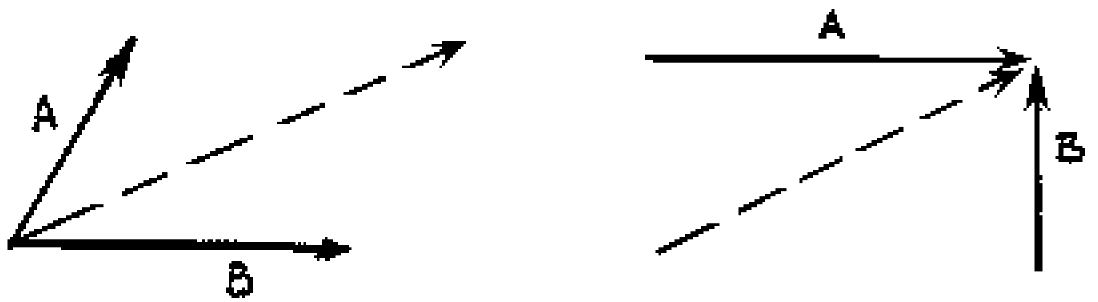
Additional Reading

Shaeffer, R. E. *Elementary Structures for Architects and Builders*. pp. 21-33.
Allen, Ed. *Grahical Statics*.

Lecture 8:

Graphic Methods of Coplanar Force Resolution

The **Parallelogram of Forces Method** is one of the graphical methods developed to find the resultant of a coplanar force system. Two or more concurrent forces can be replaced by a single resultant force that is statically equivalent to these forces.

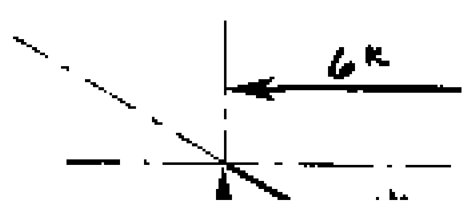


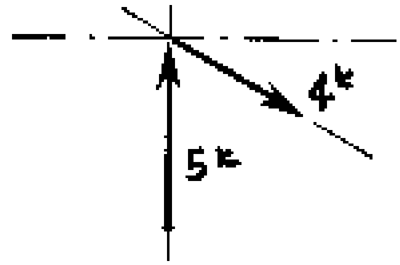
The illustration shows two vectos and their resultant. The resultant force is shown as the dashed vector. In order to resolve these forces graphically, one must first extend the lines of action of two concurrent forces until they intersect. This intersection is known as the point of origin for the system. Both forces, as well as the resultant, must ALL act either away from or toward the point of origin.

Animation illustrating how to resolve vectors with the Parallelogram method

The resultant can be represented graphically by the diagonal of the parallelogram formed by using the two force vectors to determine the length of the sides of the parallelogram. The magnitude of the resultant can be accurately measured as the scaled length of the diagonal. The resultant MUST go through the point of intersection of its components!!!

(**Remember**: graphical solutions depend upon the accuracy of the drawing. The length of each vector should be carefully scaled to equal the magnitude of the force).

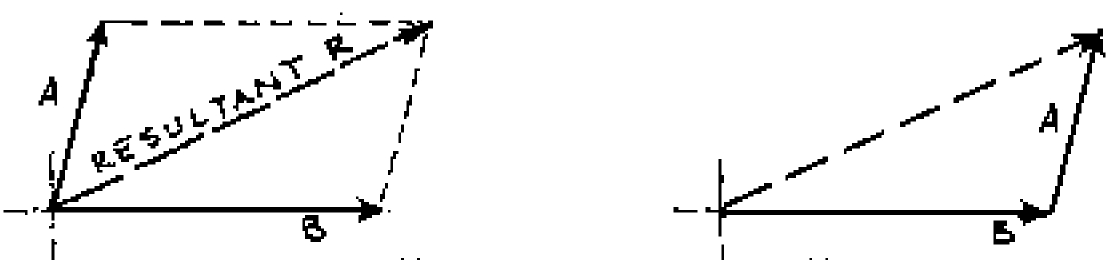




More than two non-parallel forces can be combined by successively eliminating one of the forces. Combine any two of the forces into their resultant by the parallelogram method. Combine this resultant with any of the remaining forces (or with the resultant of any of the remaining forces) until all of the forces are included. One must remember that the vectors can only be translated (or moved) along their lines of action. Two vectors (or Forces) cannot be combined (or resolved) until both of them are meeting head-to-head or tail-to-tail!

The resolution of the this system is a single vector that has a magnitude of approximately 4k with a direction of up and to the left. Try it yourself!!!

The **Triangle of Forces Method** is another graphical method developed to find the resultant of a coplanar force system. Since the opposite sides of a parallelogram are equal, a force triangle may also be found instead of using the parallelogram method. This method is quite useful because it can be simultaneously applied to any number of concurrent forces.



Parallelogram of Forces Method   Triangle of Forces Method

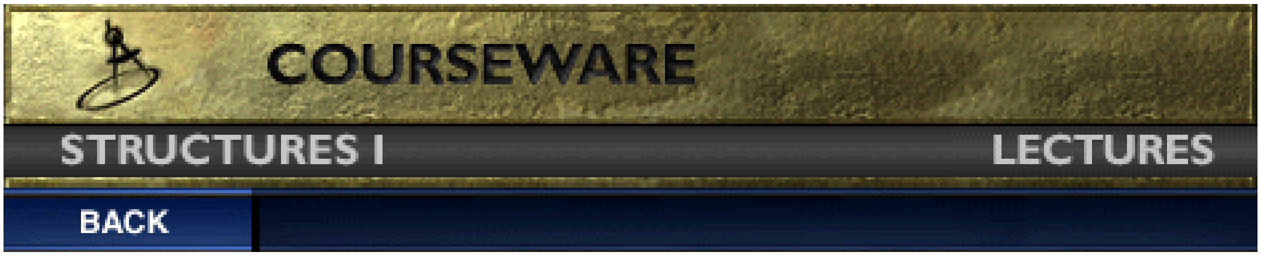
To calculate the resultant of the force system shown above, move force A so that the components A and B form a "Head-to-Tail" arrangement. The resultant R is found by starting at the tail of B (the point of intersection of forces A and B) and terminating at the head of the transposed A. Force B could have been transposed instead of force A. If this had been done, the resultant would have started from the tail of A and terminated at the head of force B.

The resultant is described by the vector's magnitude and direction. These are determined by scaling the length and angle respectively. The accuracy of these values depends upon the accuracy of the graphics.

More than two non-parallel, non-concurrent forces can be combined by successively eliminating one of the forces. Combine any two of the forces into their resultant by the triangle method, and then extend that resultant until it intersects the line of action of another force. One continues this process until all forces have been included. In this way, each one of the forces is successivly combined with the resultant of the previous triangle. One cannot simply continue to add the vectors head-to-head or tail-to-tail because the resulting lines of action would then be incorrect!

Animation illustrating how to resolve vectors with the Triangle method

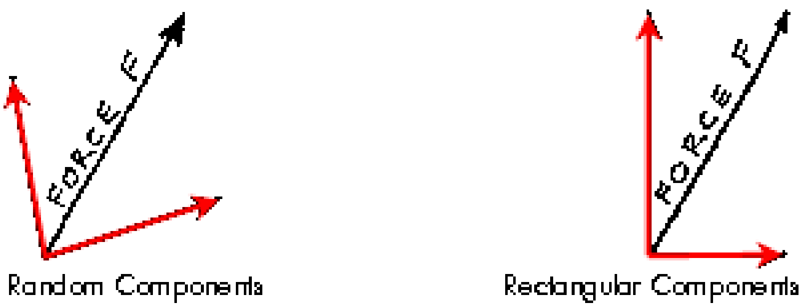
EXAMPLE PROBLEM The Triangle Method



Lecture 9:

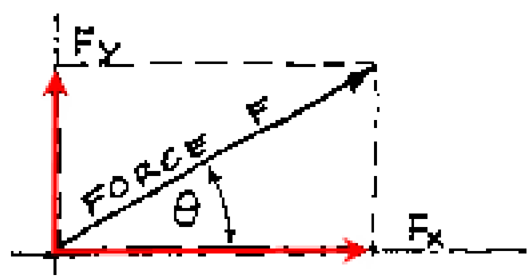
Algebraic Methods of Coplanar Force Resolution

A force can be resolved into an infinite number of combinations of components by the parallelogram method. The most useful components are the two components that are parallel to the X and Y axes. These are known as the **rectangular components**.



The solution of many problems is greatly simplified by using the rectangular components. These are usually noted as F_x, for forces parallel to the X axis, and F_y, for forces parallel to the Y axis. They are also known simply as H, for horizontal, and V, for vertical.

The rectangular components may be determined graphically, where the force is shown as a vector, or algebraically. In order to resolve a vector into its components, $F_x = F \cos ø$ or $F_y = F \sin ø$, one must know at least two items of the six geometric descriptors of a triangle (the lengths of the sides and the three angles).



Conversely, if the magnitude of the rectangular components are known, the resultant can be found with the Pythagorean Theorem, F = SQRT (F_x^2 + F_y^2). The direction of the resultant may be determined by trigonometry knowing that the tan ø is = opposite side / adjacent side.

The solutions to many problems are greatly simplified by resolving multiple forces into a resultant or by finding the rectangular components of a force system. This tool will be used extensively throughout the course.

In summary:

> **F_x = F cos ø**
> **F_y = F sin ø**
> **F = SQRT (F_x^2 + F_y^2)**
> **tan ø = F_x / F_y = opposite side / adjacent side**

The resultant of any number of concurrent forces can be found by resolving each force into its rectangular components and then adding the components algebraically. Remember, the sum of F_x = 0 and F_y = 0. The resulting numbers will be the compontents of the resultant. From these, the resultant can be found as before by using the Pythagorean Theorem in which R = SQRT (F_x^2 + F_y^2) and the formula for tan ø = F_y / F_x.

EXAMPLE PROBLEM Algebraic Method of Resolving Concurrent Coplanar Forces

Unlike coplanar, concurrent force systems, **Parallel Force Systems** cannot be completely resolved using the methods described above. Remember that every force is defined to have a magnitude, a direction and a line of action. The magnitude of the resultant of a parallel force system is equal to the algebraic sum of the components. This can also establish the resultant's direction and sense, but not its line of action. In order to find the resulatant's point of application another principle of equilibrium must be utilized: **the moment of the resultant about any point is equal to the moment of the components about the same point.** This principle is very useful for resolving multiple groupings of forces into a single force in order to simplify the calculation of reactions at the supports of a member or structure. This is best illustrated with the following example.

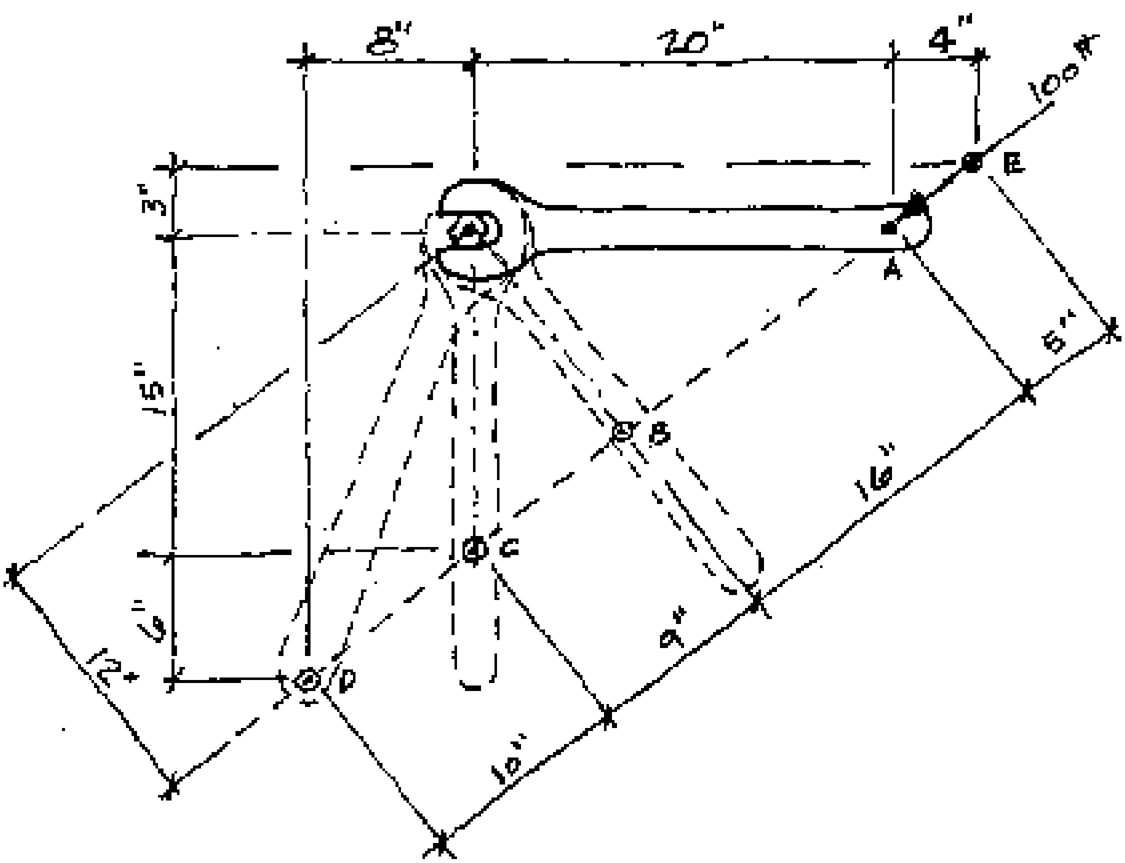
EXAMPLE PROBLEM Algebraic Resolution of a Parallel Force System

Additional Reading

Lecture 10:

Principle of Moments

The **Principle of Moments**, also known as **Varignon's Theorem**, states that the moment of any force is equal to the algebraic sum of the moments of the components of that force. It is a very important principle that is often used in conjunction with the Principle of Transmissibility in order to solve systems of forces that are acting upon and/or within a structure. This concept will be illustrated by calculating the moment around the bolt caused by the 100 pound force at points A, B, C, D, and E in the illustration.



First consider the 100 pound force.
Since the line of action of the force is not perpendicular to the wrench at A, the force is broken down into its orthagonal components by inspection. The line of action of the the 100 pound force can be inspected to determine if there are any convenient geometries to aid in the decomposition of the 100 pound force. The 4 inch horizontal and the 5 inch diagonal measurement near point A should be

recognized as belonging to a 3-4-5 triangle. Therefore, F_x = -4/5(100 pounds) or -80 pounds and F_y = -3/5(100 pounds) or -60 pounds.

Consider Point A.
The line of action of F_x at A passes through the handle of the wrench to the bolt (which is also the center of moments). This means that the magnitude of the moment arm is zero and therefore the moment due to F_{Ax} is zero. F_{Ay} at A has a moment arm of twenty inches and will tend to cause a positive moment.

F_{Ay} d = (60 pounds)(20in) = **1200 pound-inches** or 100 pound-feet

The total moment caused by the 100 pound force F at point A is 1200 pound-inches.

Consider Point B.
At this point the 100 pound force is perpendicular to the wrench. Thus, the total moment due to the force can easily be found without breaking it into components.

F_B d = (100 pounds)(12in) = **1200 pound-inches**

The total moment caused by the 100 pound force F at point B is again 1200 pound-inches.

Consider Point C.
The force must once again be decomposed into components. This time the vertical component passes through the center of moments. The horizontal component F_{Cx} causes the entire moment.

F_{Cx} d = (80 pounds)(15inches) = **1200 pound-inches**

Consider Point D.
The force must once again be decomposed into components. Both components will contribute to the total moment.

F_{Dx} d = (80 pounds)(21inches) = 1680 pound-inches
F_{Dy} d = (60 pounds)(8in) = -480 pound-inches

Note that the y component in this case would create a counterclockwise or negative rotation. The total moment at D due to the 100 pound force is determined by adding the two component moments. Not surprisingly, this yields **1200 pound-inches**.

Consider Point E.
Following the same procedure as at point D.

F_{Ex} d = (80 pounds)(3in) = -240 pound-inches
F_{Ey} d = (60 pounds)(24in) = 1440 pound-inches

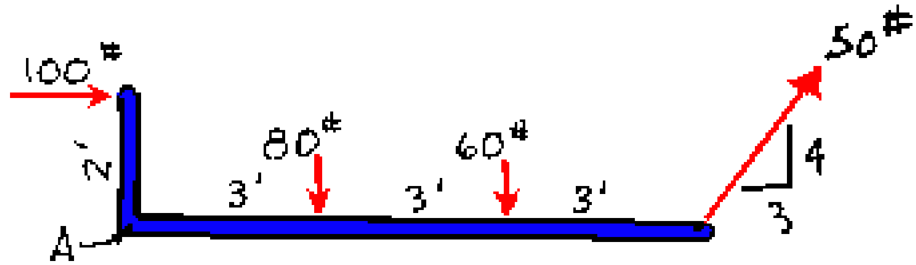
However, this time F_x tends to cause a negative moment. Once again the total moment is **1200 pound-inches**.

At each point, A, B, C, D and E the total moment around the bolt caused by the 100 pound force equalled 1200 pound-inches. In fact, the total moment would equal 1200 pound-inches at ANY point along the line of action of the force. This is Varignon's Theorem.

EXAMPLE PROBLEM Principle of Moments

Lecture 11:

Non-Concurrent, Non-Parallel Force Systems



The principles of equilibrium are also used to determine the resultant of **non-parallel, non-concurrent** systems of forces. Simply put, all of the lines of action of the forces in this system do not meet at one point. The parallel force system was a special case of this type. Since all of these forces are not entirely parallel, the position of the resultant can be established using the graphical or algebraic methods of resolving co-planar forces discussed earlier or the link polygon.

There are a number of ways in which one could resolve the force system that is shown. One graphical method would be to resolve a pair of forces using the parallelogram or triangle method into a resultant. The resultant would then be combined with one of the remaining forces and a new resultant determined, and so on until all of the forces had been accounted for. This could prove to be very cumbersome if there is a great number of forces. The algebraic solution to this system would potentially be simpler if the forces that are applied to the system are easy to break into components. The algebraic resolution of this force system is illustrated in the example problem.

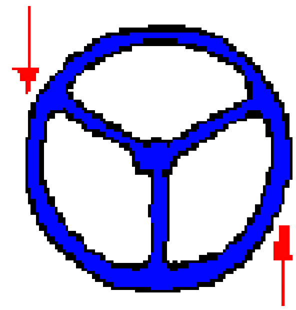
EXAMPLE PROBLEM Algebraic Resolution of a Non-Concurrent System

The Link Polygon is a graphical method to determine the resultant of non-concurrent, non-parallel force systems. It depends on an accurate graphical representation of all of the forces acting on a system. Essentially, each of the given forces is successively replaced by two components that are arranged so that one component of the force is equal in value to a component of the succeeding force. When all of the components have been combined in this way, the remaining force is the resultant of the system. This is accomplished by having a common "polar point" within the force polygon from which the components of the forces are generated. This point can be located anywhere in space, but is often chosen so that the link polygon will fit on the free body diagram.

Additional Reading

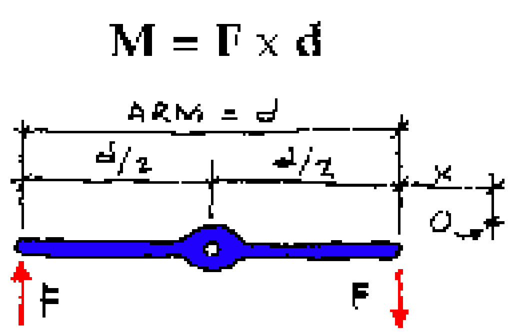
Lecture 12:

Couples



A special case of moments is a couple. A **couple** consists of two parallel forces that are equal in magnitude, opposite in sense and do not share a line of action. It does not produce any translation, only rotation. The resultant force of a couple is zero. BUT, the resultant of a couple is not zero; it is a pure moment.

For example, the forces that two hands apply to turn a steering wheel are often (or should be) a couple. Each hand grips the wheel at points on opposite sides of the shaft. When they apply a force that is equal in magnitude yet opposite in direction the wheel rotates. If both hands applied a force in the same direction, the sum of the moments created by each force would equal zero and the wheel would not rotate. Instead of rotating around the shaft, the shaft would be loaded with a force tending to cause a translation with a magnitude of twice F. If the forces applied by the two hands were unequal, there would again be an unbalanced force creating a translation of the "system." A pure couple always consists of two forces equal in magnitude.



The moment of a couple is the product of the magnitude of one of the forces and the perpendicular distance between their lines of action. **M = F x d.** It has the units of kip-feet, pound-inches, KN-meter, etc. The magnitude of the moment of a couple is the same for all points in the plane of the couple. A couple may be moved anywhere in its plane or a parallel plane without changing its external effect. The magnitude of the couple is independent of the reference point and its tendency to create a rotation will remain constant. This can be illustrated with the simple illustration of a bar with a length **d** that is pinned at its midpoint. Two parallel forces of equal magnitude, opposite in sense are applied at the ends of the bar. The magnitude of the moment generated by the couple of the forces **F**, relative to the pin in the illustration, is equal to

$$(F)(d/2) + (F)(d/2)$$

= (F)(d)

The magnitude of the couple of the forces **F** relative to point "O" is

(F)(d+x) - (F)(x)
(F)(d) + (F)(x) - (F)(x)

= (F)(d)

Again, it can be seen that the magnitude of the couple is independent of the reference location. It is always equal to (F)(d)!

The resultant of a number of couples is their algebraic sum. A couple **CANNOT** be put in equilibrium by a single force! A couple can only be put in equilibrium by a moment or another couple of equal magnitude and opposite direction anywhere in the same plane or in a parallel plane. If a single force is added to the system that balances the sum of the moments, one of the other two equations of equilibrium will not be satisfied. A couple maintains the internal equilibrium of a simple beam. The concept is very important to the further study of structural behaviour.

Additional Reading

Lecture 13:

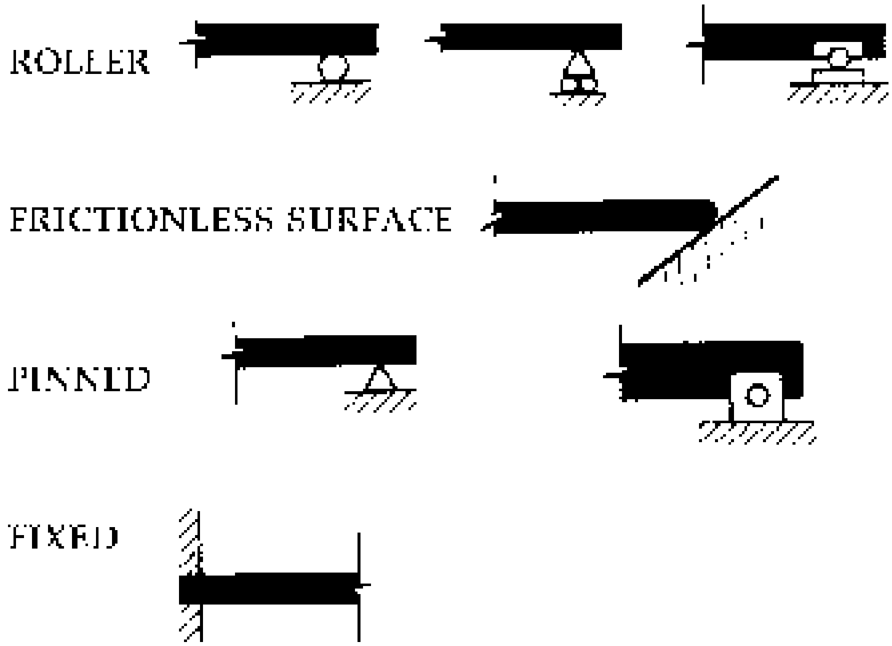
Supports

Structural elements carry their loading to other elements or the ground through connections or supports. In order to be able to analyze a structure it is necessary to be clear about the forces that can be resisted at each support. The actual behaviour of a support or connection can be quite complicated. So much so, that if all of the various conditions were considered, the design of each support would be a terribly lengthy process. And yet, the conditions of the supports is very important to the behaviour of the elements which are being supported.

In order to facilitate the analysis of a structure, it is often necessary to idealize the behaviour of a support. This is similar to the massless, frictionless pulley in physics homework problems. Even though these pulleys do not exist, they are useful to enable learning about certain issues. It is important to realize that all of the grpahical representations of supports are idealizations of a real connection. Effort should be made to search out and compare the reality with the grpahical and/or numerical model. It is often very easy to forget that the reality can be strikingly different than what is assumed in the idealization!

The four types of supports that can be found in structures are; roller, frictionless surface, pinned, and fixed. The type of support affects the forces and moments that are used to represent these supports. It is expected that these representative forces and moments, if properly calculated, will bring about equilibrium in the structural element.

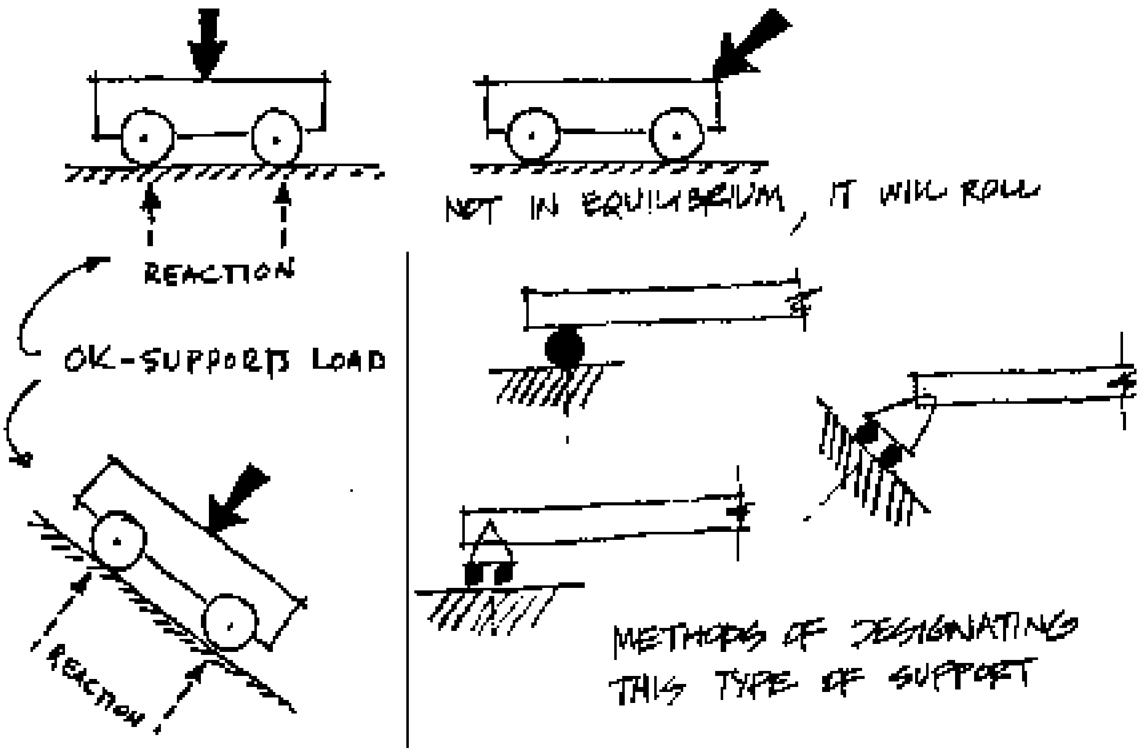
There is not a single accepted graphical method to represent each of these support types. However, no matter what the representation looks like, the forces that the type can resist is indeed standardized. Almost all supports can be assigned to one of the four types. The usual methods of graphical designation are:



These supports can be at the ends or at any intermediate points along the structural member. They are used to draw free body diagrams (FBD's) which aid in the analysis of all structural members. Each of the support types is a representation of an actual support.

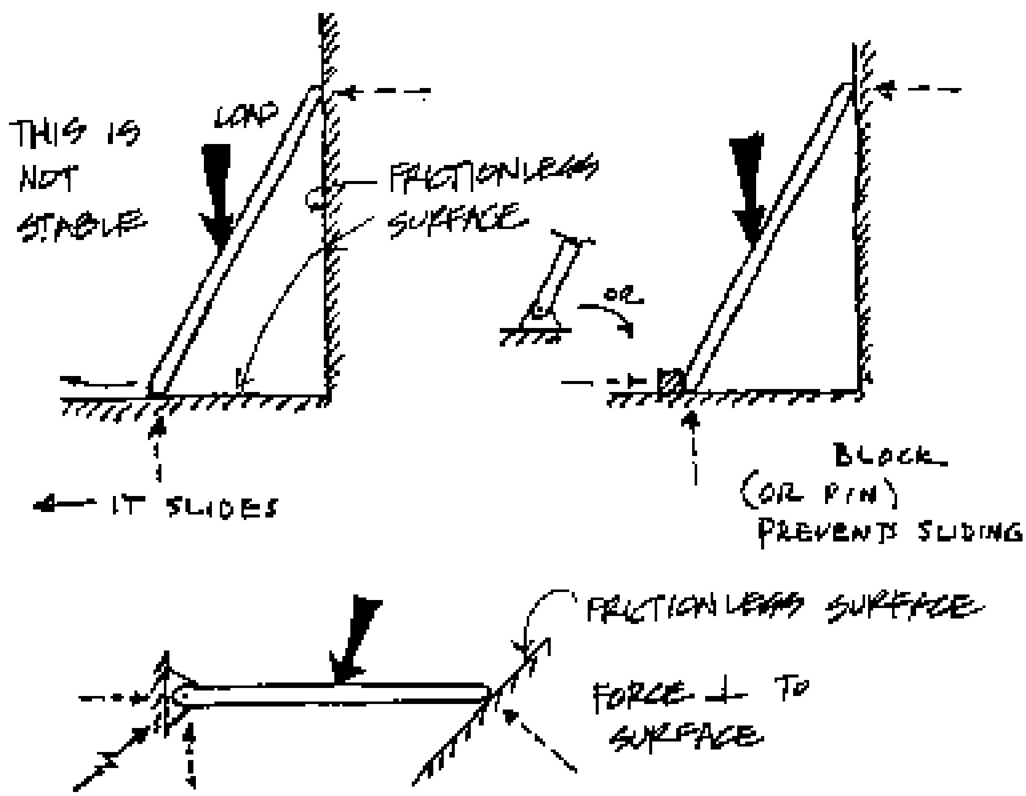
ROLLER SUPPORTS

Roller supports are free to rotate and translate along the surface upon which the roller rests. The surface can be horizontal, vertical, or sloped at any angle. The resulting reaction force is always a single force that is perpendicular to, and away from, the surface. Roller supports are always found at at least one end of long bridges so that forces due to thermal expansion and contraction are minimalized. These supports can also take the form of rubber bearings which are designed to allow a limited amount of lateral movement. A roller support cannot provide any resistance to lateral forces. The representation of a roller support includes one force perpendicular to the surface.



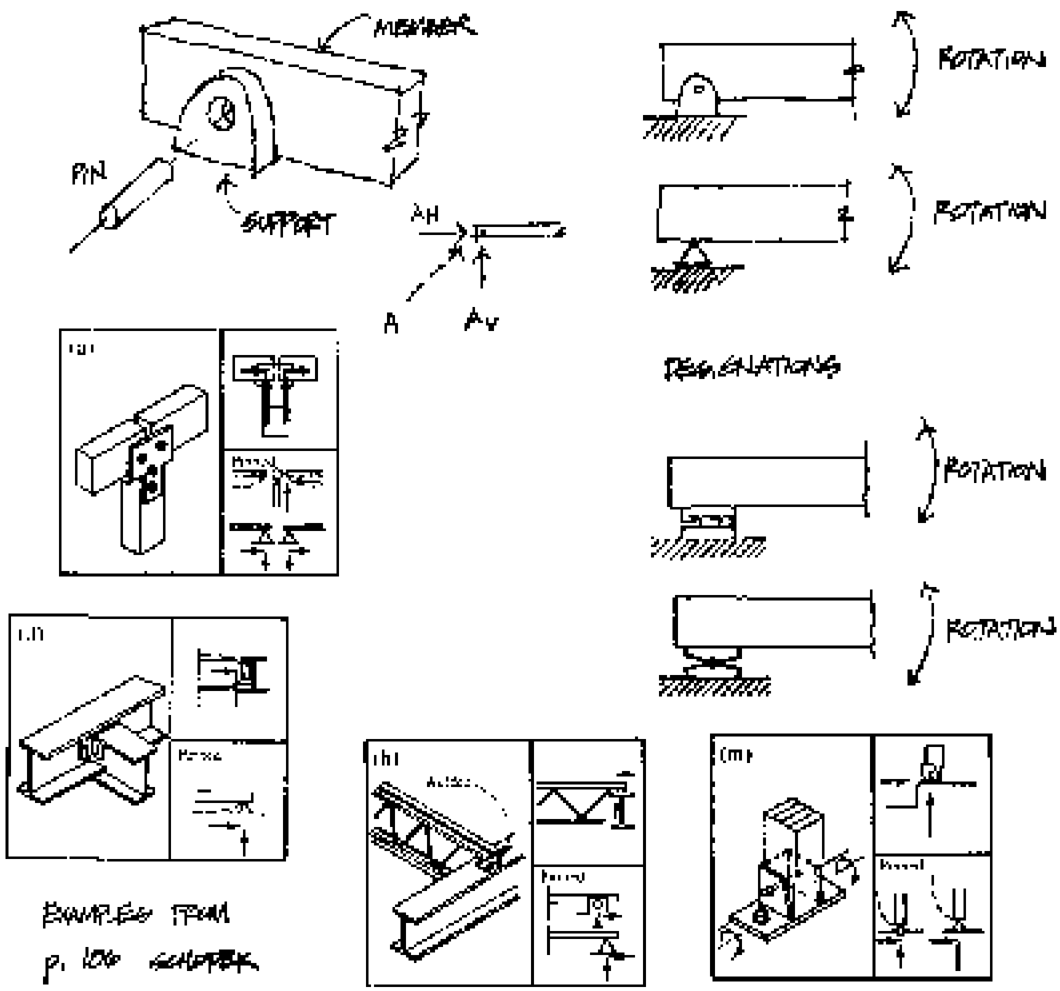
FRICTIONLESS SUPPORTS

Frictionless surface supports are similar to roller supports. The resulting reaction force is always a single force that is perpendicular to, and away from, the surface. They too are often found as supports for long bridges or roof spans. These are often found supporting large structures in zones of frequent seismic activity. The representation of a frictionless support includes one force perpendicular to the surface.



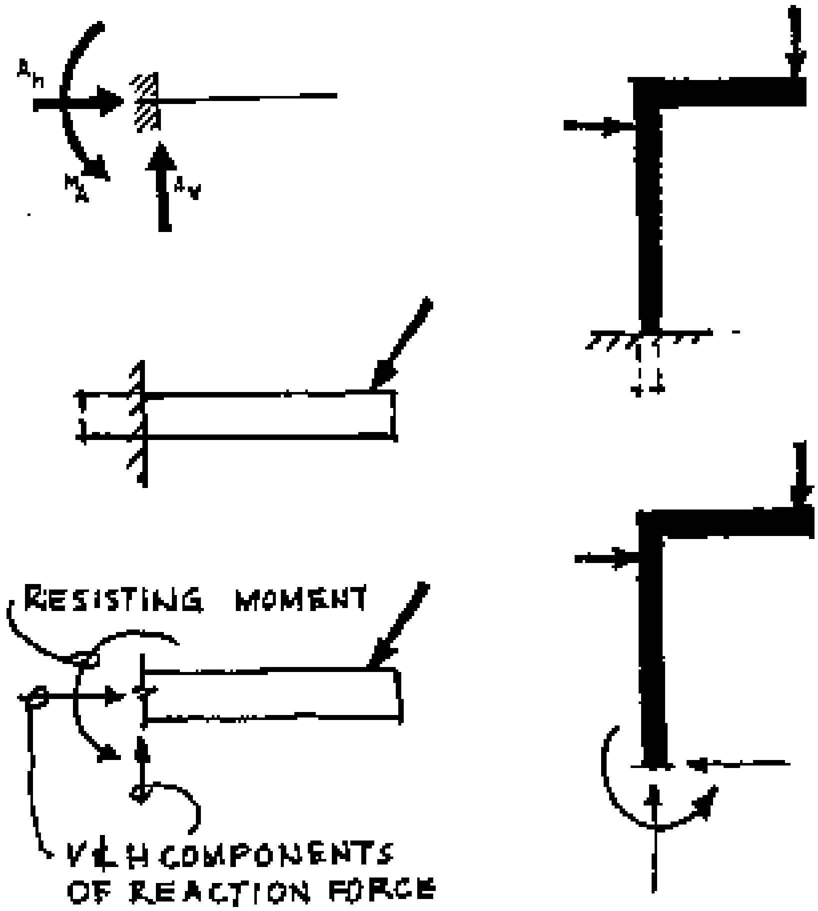
PINNED SUPPORTS

A pinned support can resist both vertical and horizontal forces but not a moment. They will allow the structural member to rotate, but not to translate in any direction. Many connections are assumed to be pinned connections even though they might resist a small amount of moment in reality. It is also true that a pinned connection could allow rotation in only one direction; providing resistance to rotation in any other direction. The knee can be idealized as a connection which allows rotation in only one direction and provides resistance to lateral movement. The design of a pinned connection is a good example of the idealization of the reality. A single pinned connection is usually not sufficient to make a structure stable. Another support must be provided at some point to prevent any rotation of the structure. The representation of a pinned support includes both horizontal and vertical forces.



FIXED SUPPORTS

Fixed supports can resist vertical and horizontal forces as well as a moment. Since they restrain both rotation and translation, they are also known as rigid supports. This means that a structure only needs one fixed support in order to be stable. All three equations of equilibrium can be satisfied. A flagpole set into a concrete base is a good example of this kind of support. The representation of fixed supports always includes two forces (horizontal and vertical) and a moment.



Additional Reading

Lecture 14:

Free Body Diagrams

One of the most useful aids for solving a statics problem is the free body diagram (FBD). A free body diagram is a graphic, symbolic representation of the body (structure, element or piece of an element) with all connecting "parts" removed. The "parts" which have been removed are all of the real physical aspects of the structure. The body is represented by a simple line. Every "part" which has been removed is replaced by the external forces representing the internal forces present where that "part" connects with the other member in the FBD. Loads, distributed or concentrated, are removed and replaced with representational force systems.

The movie below illustrated the way in which each of the loads on the structure (in this case a bench) are resolved into single loads. Each and every physical load that acts on the structure must be represented. This means that all of the laods are replaced by vectors. Even the supports are replaced by single loads.

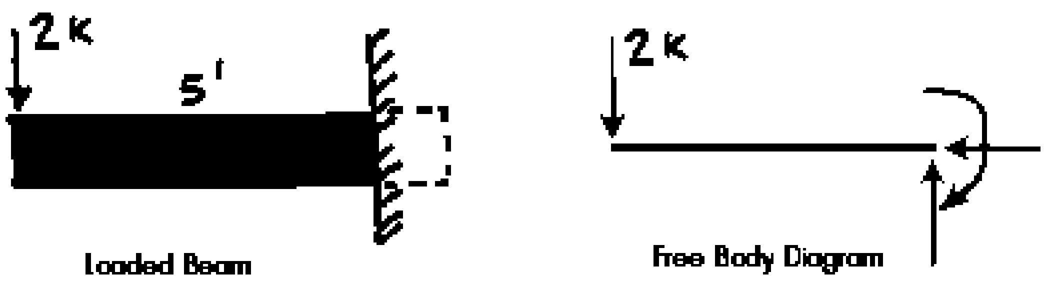
Creating a Free Body Diagram

EXAMPLE PROBLEM Reactions of a Beam

EXAMPLE PROBLEM Horizontal Components of a Reaction

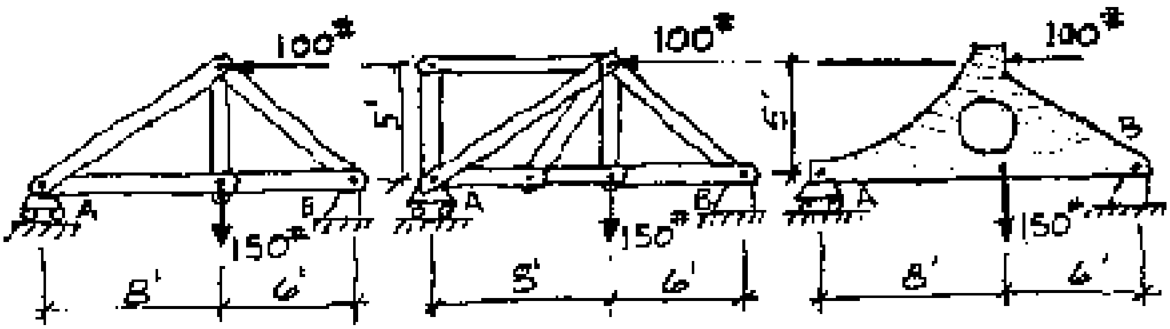
Everything that is needed to solve a force system is included on the FBD. Free body diagrams may not seem necessary in the relatively simple current applications, but as problems become more complex, their usefulness increases.

The following is the process for determining the reaction at the wall for a cantilever beam. A FBD is first drawn of the beam. Next, cut the beam free from the wall and replace the wall with the forces that were supporting the beam at the wall before it was cut free. These forces are unknown, but they are the only forces that can keep the beam in equilibrium. They are identical to the internal forces in the beam at that point before it was cut. The internal forces in the beam before it was cut free from its support are also determined when the forces which will keep, or put, the FBD in equilibrium are found.

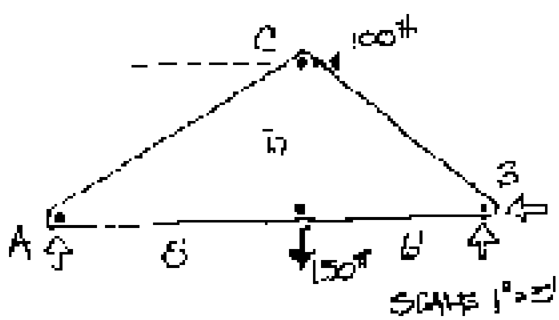


A fixed support will resist translation in all directions and rotation (moment). The FBD must show all of these directions. The principles of equilibrium can always be used to solve a FBD. In the FBD above Sum F_y = 2K and Sum F_x = 0. The 2K forces (load and vertical reaction force) cause a counter-clockwise couple of 10 K-FT which must be resisted by a moment on the end of the cut section of 10 K-FT acting in a clockwise direction.

The following are three different systems which all have one 100 pound load and one 150 pound load actin on them at exactly the same point. They are also supported in the same manner.



This is a Free Body Diagram of these three systems which has been drawn to represent the force system. Note how the internal arrangements of the three has been removed. The internal arrangement does not matter for the determination of the supporting reactions ! AND, if the supporting and loading geometries are the same, the external reactions will be the same.



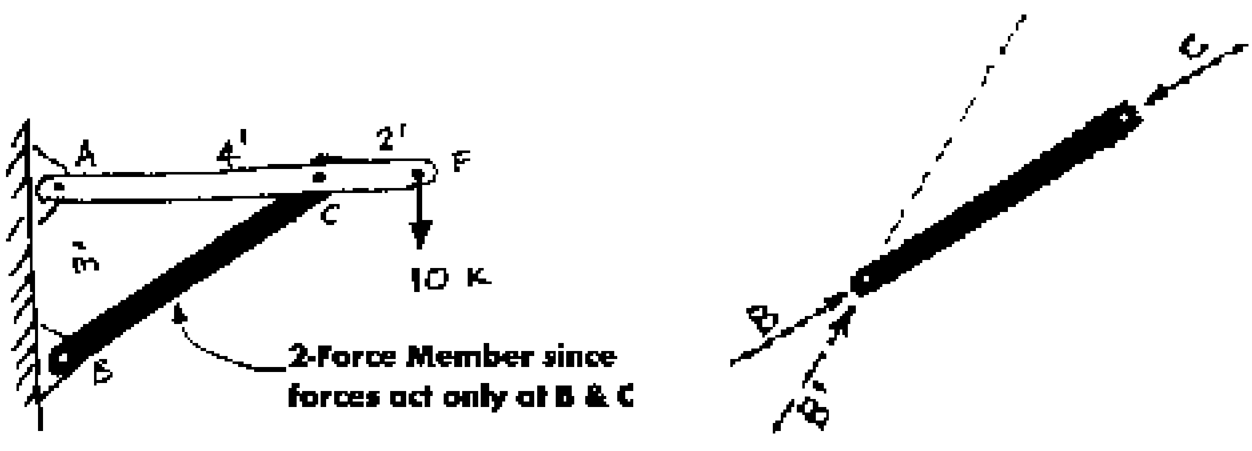
Lecture 15:

Two- and Three-Force Members

If two forces act on a body in equilibrium, they must be equal in magnitude, co-linear and opposite in sense (**the two-force principle**). A **two-force member** is a structural element which is only acted upon by two forces. The two-force principle applies to ANY member or structure that has only two forces acting on it. This is easily determined by simply counting the number of places where forces act on that member. (REMEMBER: reactions are considered to be forces!) If they act in two places, it is a two-force member.

Most, but not all, two-force members are straight. Straight elements are subjected to either tension or compression. Those members of other geometries will have bending across their section in addition to tension or compression, but the two-force principle still applies. There are NO EXCEPTIONS!!!

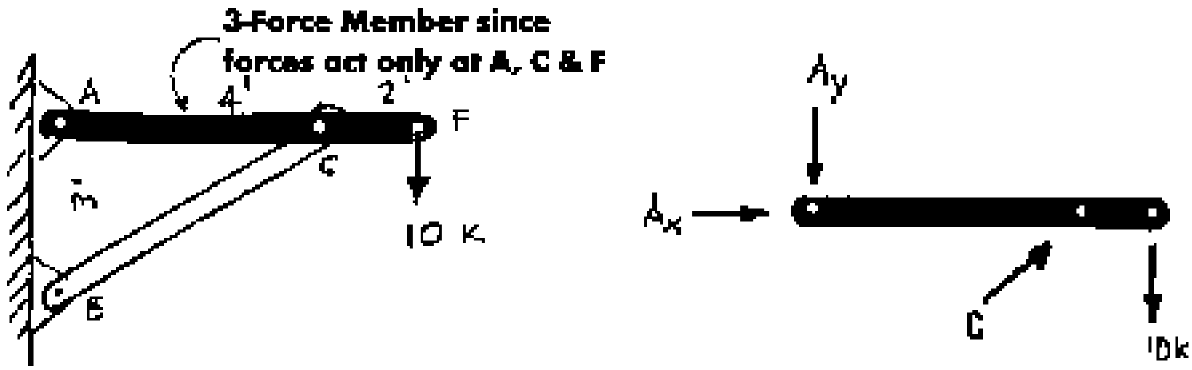
Some common examples of two-force members are columns, struts, Hangers, Braces, and Truss Members.



Let us examine the simple system shown. It could be the support for a canopy over a door. The load at point F could be a hanging lamp. All of the joints are considered to be pinned. If member BC is isolated, it can be seen that it has forces acting at only points C and B. This means that it is a two-force member. The line of action of the force at point C must also pass through point B; similarly, a force at point B must also pass through point C. If the force at B did not pass through point C (B' in the diagram), the force would cause a moment about point C and equilibrium would not be possible. Because the two forces are equal in magnitude, co-linear and opposite in sense, two-force members act only in pure tension or pure compression. Supports such as cables tend to work well as two force members.

If three non-parallel forces act on a body in equilibrium, it is known as a **three-force member**. This often refers to elements which have a single load and two reactions. If a three-force member is in equilibrium and the forces are not parallel, they must be concurrent. Therefore, the lines of action of all three forces acting on such a member must intersect at a common point; any single force is the equilibrant of the other two forces. These members usually have forces which cause bending and sometimes additional tension and compression.

The most common example of a three-force member is a beam.

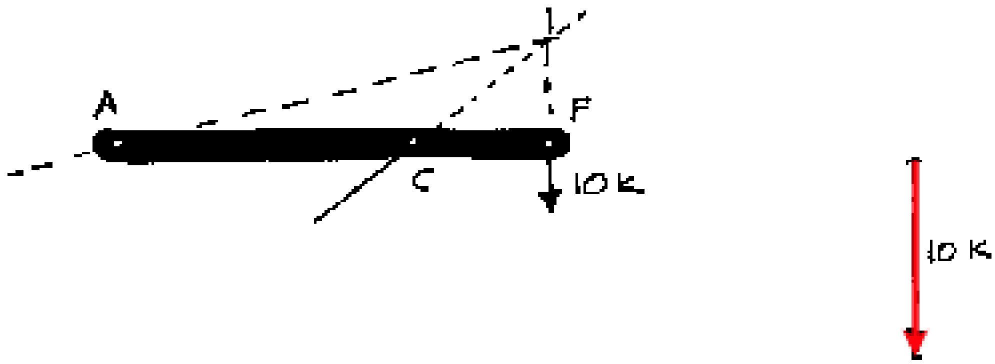


If one isolates member AF in the pin connected frame to the right, one sees that it has forces acting at three points: A, C, and F. The free body diagram of the system can be seen in the diagram below. The magnitude and the line of action of the force at F, 10 Kips, is known. The line of action of the force at point C is known because it must be equal and opposite to the force C of the two-force member CB. The line of action of the forces at point F and point C intersect at X. The line of action of the force at point A must also go through points A and X. (Why is this?)

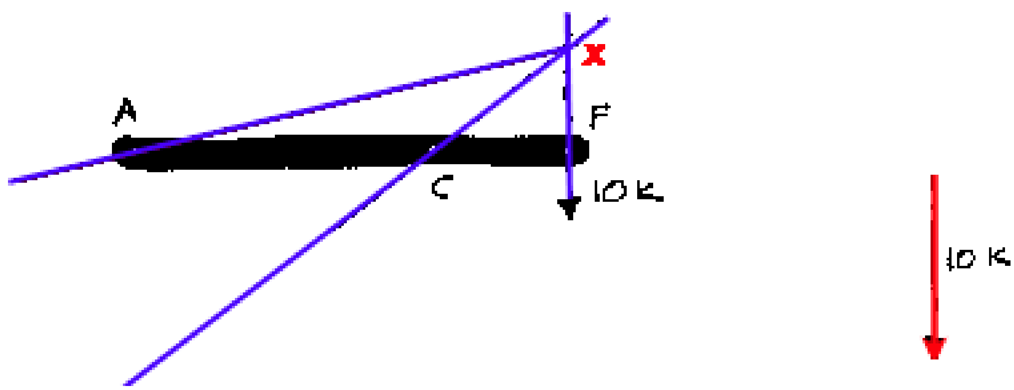
The lines of action of the reactions at points A and C have now been determined. The problem of establishing their sense and magnitude remains. The sense of these forces can be established intuitively in this example, but this is not always the case. The **Three Force Principle**, demonstrated in a step-by-step manner, will show how simple it is to establish both the sense and the magnitude of the reactions of a system of three forces:

1. Draw the load vector to some convenient scale.
2. Select either of the other forces and draw a line parallel to that force through the head of the load vector.
3. Through the tail of the load vector, draw a line parallel to the remaining force. (If these lines are extended far enough in each direction they form a closed polygon).
4. Apply arrows on the vectors so that they are now connected head-to-tail. The force polygon is now complete; the arrows show the sense and the vectors can be scaled to determine the

magnitude.



Had it been assumed that the line of action of the reaction force going through point A had taken a direction other than through point X, the system would not be a concurrent force system. Although it could be in force equilibrium, it would not be in moment equilibrium because the summation of the moments about ANY point would no longer be zero. This can be seen below.



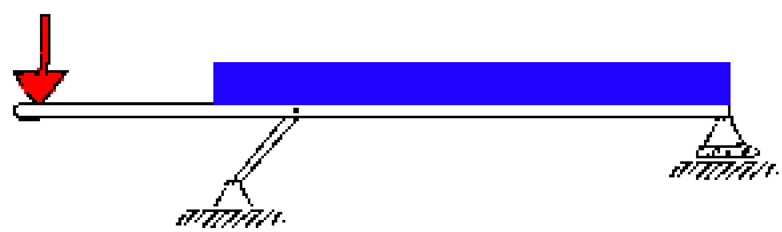
EXAMPLE PROBLEM Three-Force Members

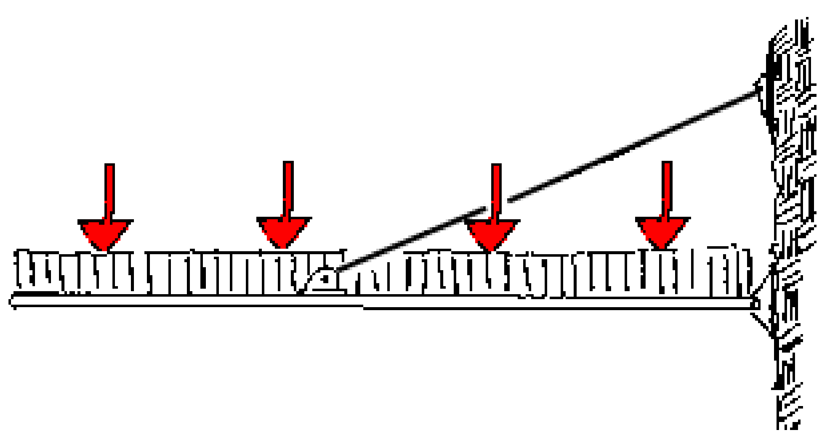
Lecture 16:

Multiple Force Members

The term, *three-force member*, is often incorrectly applied to members with more than three forces. Any member which is subjected to more than three single concentrated loads should be identified as a **Multiple Force Member**. The "extra" forces acting upon a multiple force member can be replaced until the total number of forces is reduced to three; the resultant force of the loading and the two reactions. This means that distributed loads must always be replaced so that only one resultant force is applied to the member. Then, all of the other forces must be replaced so that there is only one resultant representing the external loading. The three non-parallel forces must be concurrent so that the three-force principle then applies.

The illustrations are two examples of multiple force members:





The loading of the first image would be resolved in the following manner. The resultant of the distributed load would be determined. This would act at the mid-point of the width of the load. The resultant of this force and the concentrated load acting at the cantilevered end of the beam would be combined into one load by summing moments about the right-hand support. This system would then be reduced from multiple force member to a three force member.

The second system could represent an entrance canopy to a hotel that also has four large spot lights sitting upon it. These six loads must also be combined into one load resultant before the member can be considered a three force member.

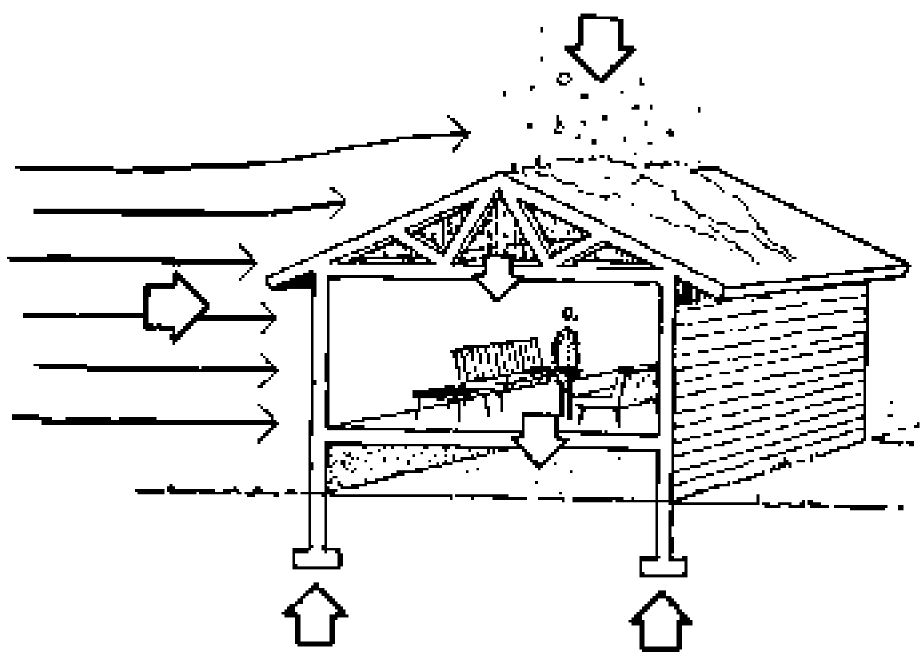
EXAMPLE PROBLEM System Reactions

Lecture 17:

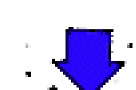
Primary Loads

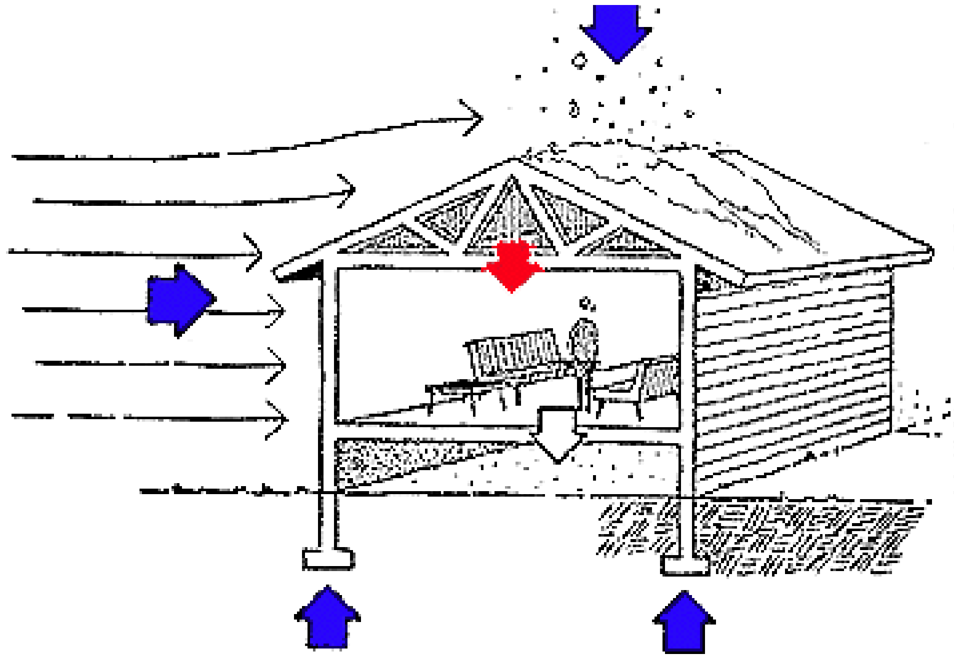
The loading of a building structure can take on a wide variety of forms. In many cases the exact loading will not fit neatly into a specific category. Yet, loads can usually be considered to be Primary or Secondary. Primary loads are discussed below. Secondary loads are those loads due to temperature changes, construction eccentricities, shrinkage of structural materials, settlement of foundations, or other such loads. Despite the fact that each and every load and loading combination should be considered in order to reduce the chance of structural failure, the determination of the loading remains a statistical exercise. Each and every load cannot be foreseen; thus, it is critical to determine the worst case that is reasonable to assume to act upon the structure. The sources of primary loading include the materials from which the structure was built, the occupants, their furniture, and various weather conditions, as well as unique loading conditions experienced during construction, extreme weather and natural catastrophes.

Primary Loads are divided into two broad categories according to the way in which they act upon the structure or structural element. These are DEAD LOADS and LIVE LOADS. When considering the possible combinations of these two categories of loading, the odds of certain loads occuring simultaneously are assumed to be null. One such combination would be heavy snow, a typhoon, a raging fire and an earthquake. It is possible that two of the first three could occur as the earth quakes, but not that all four would be present at the same time. Thus, one need only consider reasonable loading combinations.



DEAD LOADS

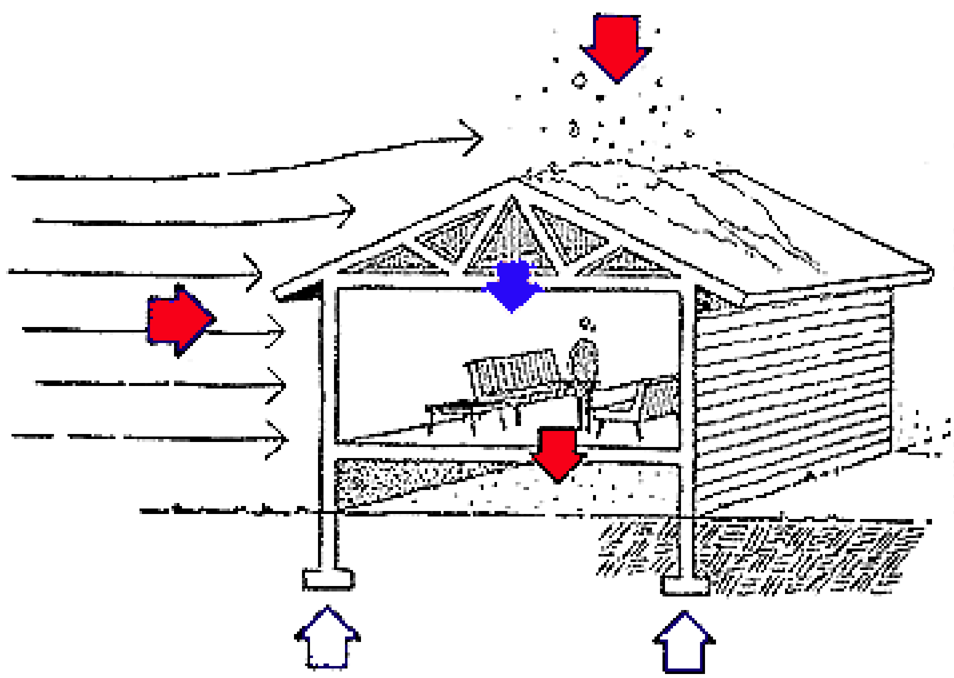




Dead Loads are those loads which are considered to act permanently; they are "dead," stationary, and unable to be removed. The self-weight of the structural members normally provides the largest portion of the dead load of a building. This will clearly vary with the actual materials chosen. Permanent non-structural elements such as roofing, concrete, flooring, pipes, ducts, interior partition walls, Environmental Control Systems machinery, elevator machinery and all other construction systems within a building must also be included in the calculation of the total dead load. These loads are represented by the red arrow in the illustration.

The magnitude of the dead loads of a building can usually be determined with only a 5% margin of error. Properties of building materials are often tabulated and published by authors of textbooks and by the manufacturers. It is always very important to be sure to keep up-to-date on the changes in building materials. Properties of construction materials often vary due to the rapidly changing marketplace. The load due to these materials is often expressed as kN/m^3 or lbs/ft^3. These are normally converted to load/area or load/length for further consideration.

LIVE LOADS



Live Loads are not permanent and can change in magnitude. They include items found within a building such as furniture, pianos, safes, people, books, cars, computers, machinery, or stored materials, as well as environmental effects such as loads due to the sun, earth or weather. Wind and earthquakes loads are put into the special category of lateral live loads due to the severity of their action upon a building and their potential to cause failure.

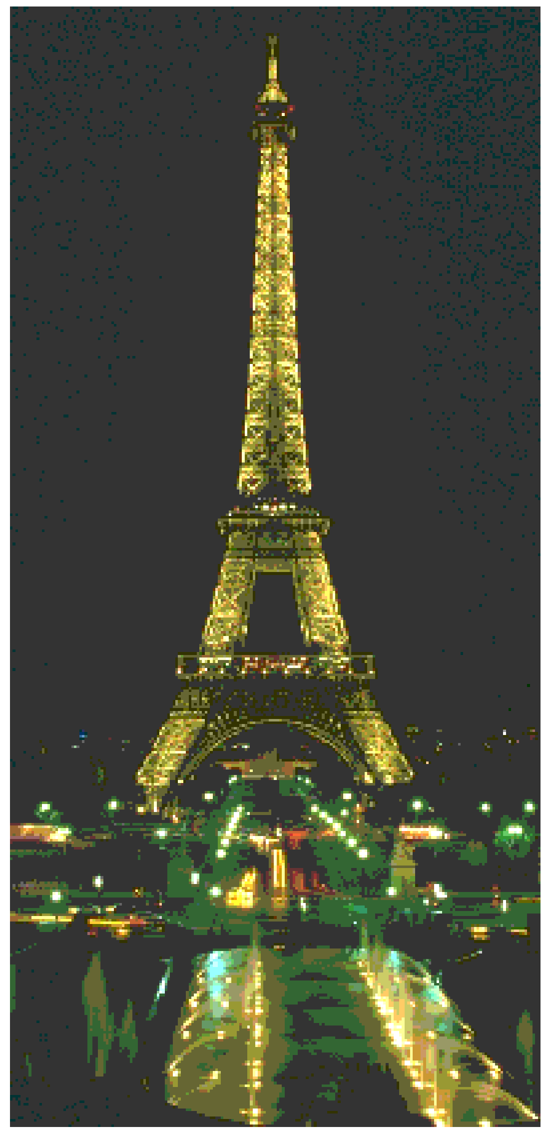
Most buildings have a working life that extends beyond the initially perceived use of the building. Thus, it is almost impossible to predict all of the potential uses that any given structure will experience before it is demolished. If, and when, a building is to be used for a purpose other than its original design, the capacity of the building for its new use must be determined. Since the body of knowledge about the behaviour of buildings is always increasing, a building which might have been designed according to the latest loading information in one year, might not satisfy the requirements a few years later. This has especially been the case with the effects of earthquake loading.

The magnitudes of live loads are difficult to determine with the same degree of accuracy that is possible with dead loads. The probable maximum value of live loads has been determined by research and is included in national building codes. These are usually a minimum design load per area. Building codes also provide for load reductions under certain conditions. As an example, full live loads will not occur on every floor of a multi-story building at the same time. Therefore, the design live load for some of the columns and the foundation can be reduced. Building codes around the world do not concur on the magnitude of the appropriate design live load values. It is critical that the designer take the time to determine the values set down in the local building codes. These are legal documents and MUST be followed.

Table 17-1. Common Unit Weights of Building Materials
Table 17-2. Common Floor Loads

Lecture 18:

Lateral Loads



Most lateral loads are live loads whose main component is a horizontal force acting on the structure. Typical lateral loads would be a wind load against a facade, an earthquake, the earth pressure against a beach front retaining wall or the earth pressure against a basement wall. Most lateral loads vary in intensity depending on the building's geographic location, structural materials, height and shape. The dynamic effects of wind and earthquake loads are usually analyzed as an equivalent static load in most small and moderate-sized buildings. Others must utilize the iterative potential of the computer. The design wind and earthquake loads on a building are substantially more complex than the following brief discussion and simple examples would indicate. The Uniform Building Code describes the design wind load determination in more detail for the various parts of the United States.

WIND LOADS
The most common lateral load is a wind load. The Eiffel Tower is one example of a building which has a structure that was designed to resist a high wind load. Wind against a building builds up a positive pressure on the windward side and a negative pressure (or suction) on the leeward side. Depending upon the shape of the structure it may also cause a negative pressure on the side walls or even the roof. The pressure on the walls and roof is not uniform, but varies across the surface. Winds can apply loads to structures from unexpected directions. Thus, a designer must be well aware of the dangers implied by this lateral load. The magnitude of the pressure that acts upon the surfaces is proportional to the square of the wind speed.

Wind loads vary around the world. Meteorological data collected by national weather services are one of the most reliable sources of wind data. Factors that effect the wind load include the geographic location, elevation, degree of exposure, relationship to nearby structures, building height and size, direction of prevailing winds, velocity of prevailing winds and positive or negative pressures due to architectural design features (atriums, entrances, or other openings). All of these factors are taken into account when the lateral loads on the facades are calculated. It is often necessary to examine more than one wind load case.

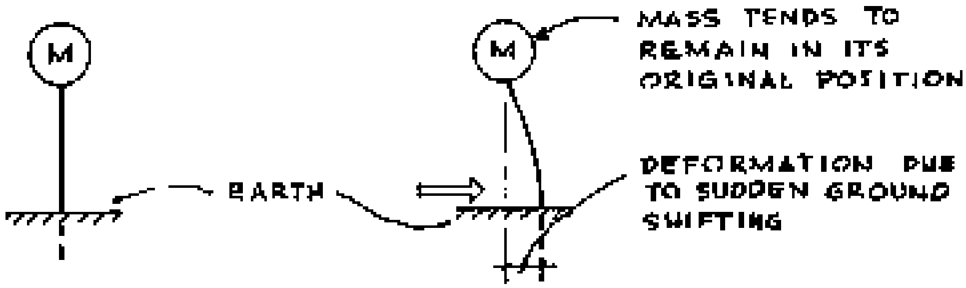
For this course, it will be assumed that wind loads, as well as the pressure they develop upon wall and roof elements, are static and uniform. They actually not only pound a structure with a constantly oscillating force, but also increase as a building increases in height. The loading of a tower can be very roughly approximated by an evenly distributed load. It is a vertical cantilever. The applet below allows you to investigate the variables which influence the structural behavior of a tall, thin tower. It does not represent actual methods of calculating the total wind force on a tall building. It is intended to demonstrate the interaction between the variables of the equations which govern the structural behavior.

EARTHQUAKE LOADS

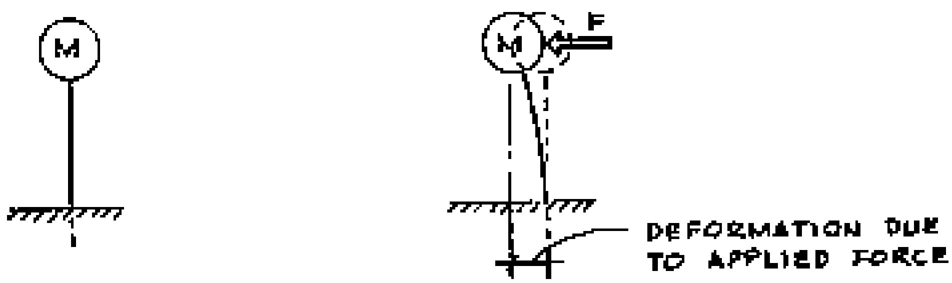
Earthquake loads are another lateral live load. They are very complex, uncertain, and potentially more damaging than wind loads. It is quite fortunate that they do not occur frequently. The earthquake creates ground movements that can be categorized as a "shake," "rattle," and a "roll." Every structure in an earthquake zone must be able to withstand all three of these loadings of different intensities. Although the ground under a structure may shift in any direction, only the horizontal components of this movement are usually considered critical in a structural analysis. It is assumed that a load-bearing structure which supports properly calculated design loads for vertical dead and live loads are adequate for the vertical component of the earthquake. The "static equivalent load" method is used to design most small and moderate-sized buildings.

The lateral load resisting systems for earthquake loads are similar to those for wind loads. Both are designed as if they are horizontally applied to the structural system. The wind load is considered to be more of a constant force while the earthquake load is almost instantaneous. The wind load is an external force, the magnitude of which depends upon the height of the building, the velocity of the wind and the amount of surface area that the wind "attacks." The magnitude earthquake load depends up the mass of the structure, the stiffness of the structural system and the acceleration of the surface of the earch. It can be seen that the application of these two types of loads is very different.

This movie is a representation of the movement of a free standing water tower in an earthquake. It can be seen that the as the ground moves, the initial tendency is for the water tower to remain in place. The shifting of the ground is so rapid that the tower cannot "keep up."



After a moment, the tower moves to catch up with the movement of the ground. The movement is actually an acceleratoin. From Newtonian Physics, it is know that an applied force=mass x acceleration. Thus, the force which is applied to the water tower depends upon the mass of the tower and the acceleration of the earth's surface.



The force in this last diagram may be thought of as the "equivalent static load" for which the structure would be designed. This idealized situation demonstrates a concept; it requires modification for actual buildings. These modifications account for building location, importance, soil type, and type of construction. This movement can also be seen in the following movie of lateral earth movement. Note how the mass slowly reacts to the movement of the earth. Eventually, the bending strength of the stem of the tower would be exceeded and it will fail.

Simulation of a water tower in an earthquake

It remains very difficult to imagine the destruction which can be wrought by an earthquake. The lessons learned from the Los Angeles Earthquake of 1994 helped structural designers change design strategies. The following illustrations are snapshots from LA:

EXTERNAL LINK comments

EXTERNAL LINK garages

EXTERNAL LINK a pkg garage

EXTERNAL LINK structural bay

EXTERNAL LINK Old Federal Bldgg

EXTERNAL LINK Northridge Mdws

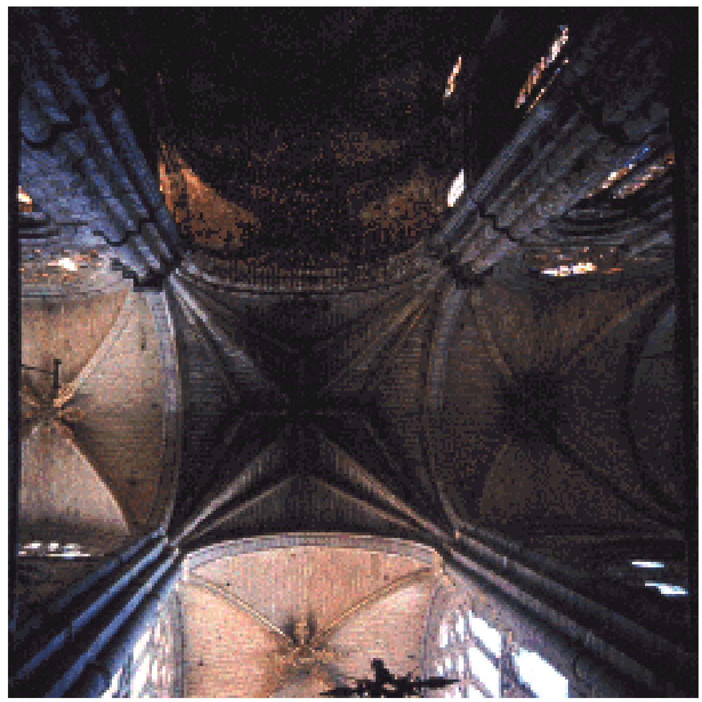
Fluid and Earth Pressure Loads

Liquids produce horizontal loads in many structures. The horizontal pressure of a liquid increases linearly with depth and is proportional to the density of the liquid. This is similar for earth pressures. These last are a bit more complex in that the load due to earth pressure varies with its depth, any surcharge, the type of soil and its moisture content. The design live load for this soil pressure must not be less than that which would be caused by a fluid weighing 30 pcf.

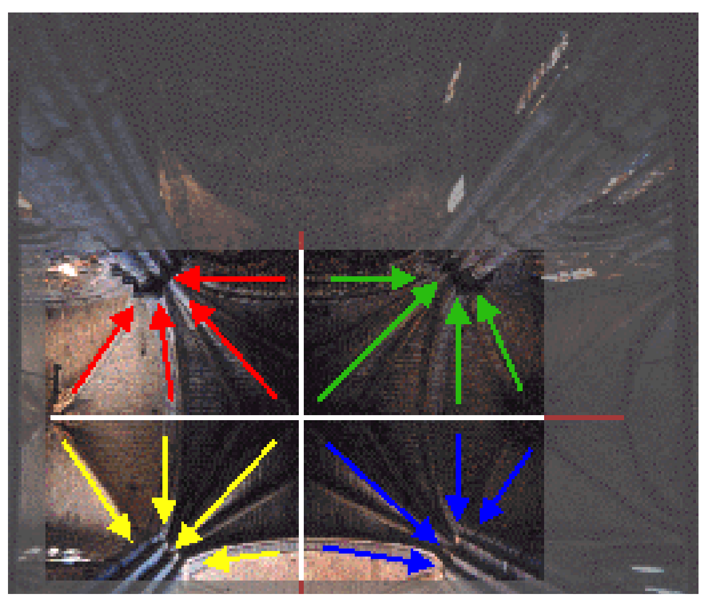
Lecture 19:

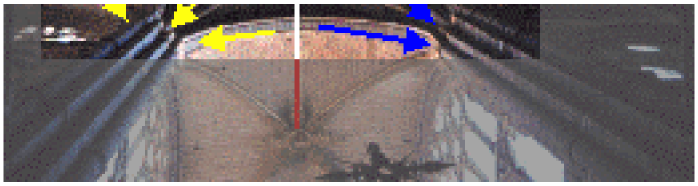
Load Distribution

Gravity Load Distribution
Architects must consider not only what loading conditions might effect a structure, but also how those loads are transfered from their point of origin through the structure to the ground. Architectural design decisions concerning the structural framing system will dictate this flow of forces. The most direct path from the point of loading to the ground will create the most efficient structure. The continuity of this line of transfer is essential. However, this direct path is not always possible. As a matter of fact, due to the nature of live loads this path is constantly changing!

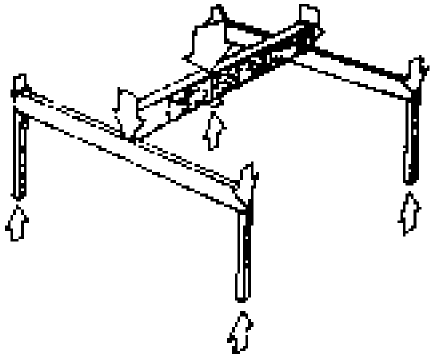


The arrows indicate the load path. >>>Under re-construction<<<

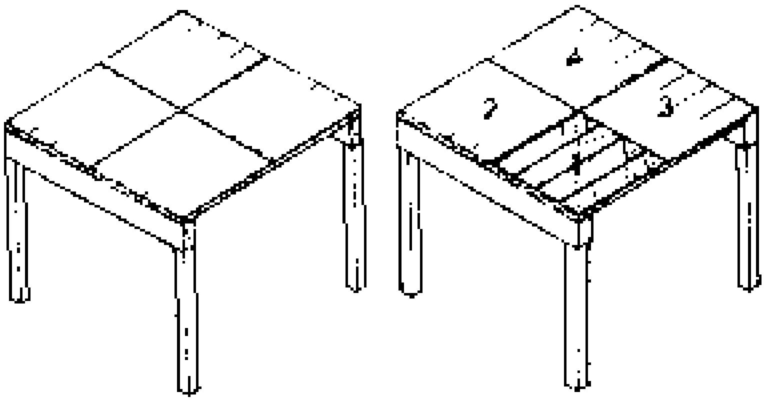




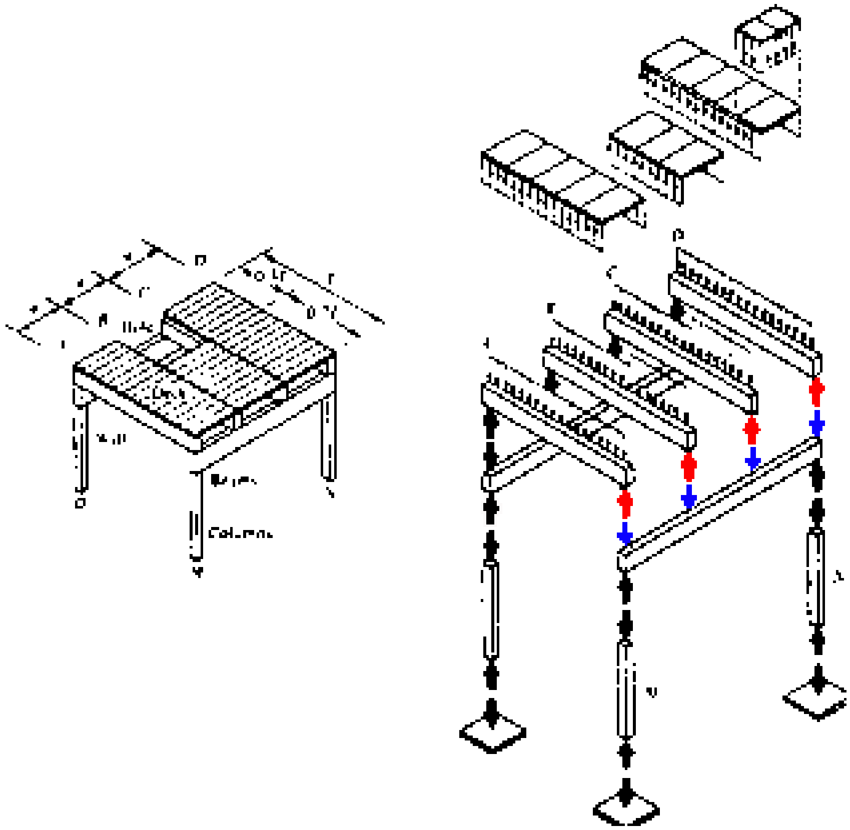
The load at the midspan of the joist of the frame is illustrated as a large arrow pointing down. This load is split into two parts, with a proportional part going to each end of the beam. This in turn becomes a point load at the midspan of the smaller beams. They transfer the load to the earth. The load on each column in this case is only one quarter of the initial load. This is one method of distributing a load.



The second drawing illustrates a wooden floor of thick planks that span between two beams. Each of the four squares of the flat surfaces is the tributary area of the beam below it. A **tributary area** of a structural element (such as a joist, beam, column, or wall) is the area that contributes to the loading of that specific element. In this case, the plans transfer their loads and pass them on to the two beams. These beams can only transfer their loads equally to the two columns which support them. Thus, the tributary area can be drawn by determining the supporting characteristics of the horizontally spanning members.



This is again illustrated below for a simple frame structure. The loads are gathered by each structural element and passed on to that element's supporting elements. These supports in turn pass their loads to the next supporting element until the original load has been transferred all the way to the earth.



The load distribution pattern and the explicit summation of the loads can have a direct effect upon the size of the elements. Load distribution often causes unequal loading of the vertical supporting members. This may or may not be indicated by the designed form of that element.

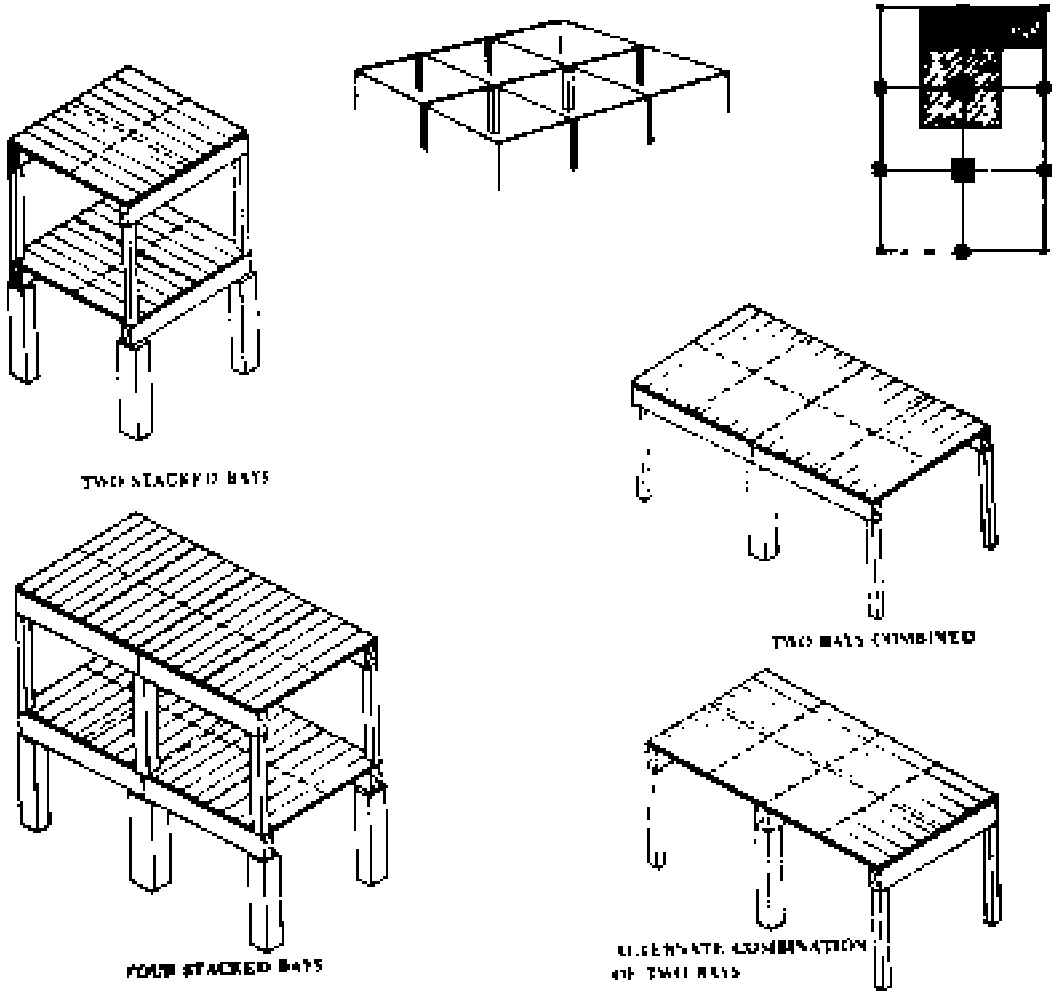


Illustration from STRUCTURE by Corkill, pp 75, 77 & 175

EXAMPLE PROBLEM [Load Distribution]

WIND LOAD DISTRIBUTION

The essence of wind load distribution through a building from one building component to another, and how the building resists this load can be investigated by assuming a uniformly distributed wind load acting on any one face of the building at any one time. Normally a wind load design requires a separate analysis of wind from two perpendicular directions, such as wind from the north or south and then from the east or west.

Consider a one story, flat roofed, rectangular, wood framed building without a parapet. The wall receiving the direct pressure from the wind distributes the top half of its horizontal wind load to the roof and the bottom half to the foundation; it acts as a vertical beam with the roof and foundation acting as simple horizontal supports. The part of the load going to the foundation is distributed from the foundation to the ground. The portion of the load going to the roof tends to cause the roof to move laterally; this lateral movement is resisted by the end walls. The movement of the end walls is prevented by their connection to the foundation.

Since winds of 80 to 110 mph will create pressure on the side walls of buildings in the range of 15 to 30 psf, design forces of the wind will be considered between 20 or 25 psf (static load) in this course.

Additional Reading

Lecture 20:

Resolving Distributed Loads

Many structural members are subjected to distributed loads rather than concentrated loads. However, when solving for their reactions, it is often convenient to replace the distributed loads with an equivilent concentrated load(s) .

The most common type of distributed load is the uniform load, in which a load of constant magnitude is applied along a length of a beam or an area of a surface. The resultant of these distributed loads has the same magnitude as the area of the distributed load and acts through the center of gravity, or the centroid, of the load area. This point is located at the intersection of the diagonals of a rectangular, or uniform, load area.

EXAMPLE PROBLEM Distributed Loads

Not all distributed loads are of a constant value. Some examples of loading conditions that may cause triangular loads are:

- the load on a lintel in a masonry wall due to the arching of the masonry;
- the load on a vertical surface below grade due to soil or water pressure;
- the load on the vertical surface of a structure due to wind loading.

These triangular loads can be replaced by a single concentrated load which acts through the centroid of the distributed load and acting in the same direction. The centroid of a triangular load area is found at the intersection of its medians which always equals 1/3 of the height above any of its bases.

EXAMPLE PROBLEM Triangular Loads

Triangular loads can be caused by either soil or water pressure. Note that it constantly increases with depth! The magnitude of the greatest load that is applied is simply the unit weight of the material multiplied by the deepest point in question.

There are cases where a uniformly distributed load combines with a triangularly distributed load to create a trapezoidal load. This loading condition is most easily solved by separating the triangular and uniform loads and calculating separate resultants for each. These resultants can then be treated as a series of concentrated loads or transformed into a single resultant for the system.

EXAMPLE PROBLEM Trapezoidal Loads

Lecture 21:

Resolution of Multiple Loads

To come....

Further Reading

COURSEWARE
STRUCTURES I LECTURES
BACK

Lecture 22:

Strength of Materials

A structural system is not only effected by external conditions, but also by the properties and behavior of the materials which comprise it. These also determine the nature of the system's reaction(s) to external forces. The study of **Strength of Materials** is concerned specifically with the following issues:

1. the internal forces of a member caused by the external forces acting on that member or system.
2. the changes in dimension of a member caused by these forces.
3. the physical properties of the material in the member.

Statics is the study of the behavior of rigid bodies at rest as they are acted upon by external forces. Although most of these bodies are not absolutely rigid, the assumption of rigidity is valid for the purpose of determining the reactions of the system. Actually, every material will deform under a load. Even a concrete slab deforms microscopicly when a person walks on it. Some deformations in a structure can be detrimental to the overall system's performance, while others might only be an issue of comfort. The recognition of the relative importance of these deformations will be an important part of the study of structures.

External loads on a structural system create resisting forces within all of the members that form the load path from the point of the application of the load to the ground beneath the foundation. This internal resistence exists within every member and joint included in the load path and are known simply as the **internal forces** acting on a member.

Some of these forces have already been examined: the connection between a beam and its support, and the connection of a two-force member to a three-force member in a pin-connected frame. The internal forces within a beam were demonstrated by cutting a beam and drawing a FBD. These internal forces were required at the cut section to put the beam back into equilibrium. The forces and moments that were examined were applied externally to the end of that cut section; they were exactly equal to the internal forces and moments.

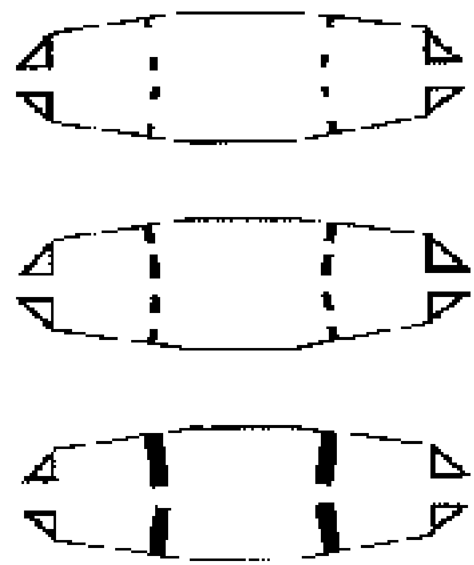
The distribution of the internal forces on the cross-sectional area of a member may or may not be uniformly distributed; it is dependent on the loading condition, the type of member, and how it is supported.

Further Reading

Chapter 3. Mainstone, Rowland J. *Developments in Structural Form.* Allen Lane/Penguin Books. Middlesex, England. 1975.

Lecture 23:

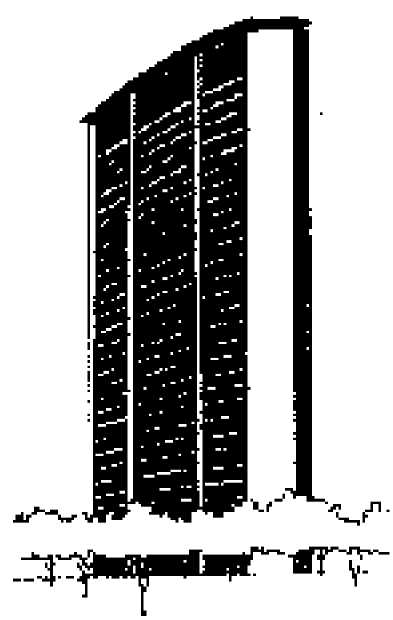
Stress and Stress Prisms



The internal forces of each member of a structural system are distributed in a particular way across that member's cross-sectional area. **Stress** is a measure of the intensity of this force on a single unit of area. Common units of measure of stress are KN/m^2, N/mm^2, #/in^2, #/ft^2, or kips/in^2. The distribution of the stress may be constant across a cross-sectional area, or it may be variable. It can vary due to the loading conditions, the material or geometry of the structural member.

Structural members are subjected to a variety of types of stress. They can be divided into primary and secondary stresses. Primary stresses are Compression, Tension, Torsion and Shear. Secondary stresses are the result of deformations of the structural system, or member, which cause additional loads within each member to occur (For example, a beam expanding in the summer sun pushing on a fixed column). These stresses will be discussed in more detail in a later lecture. Initially, the focus will be on simple tension and compression stresses for members loaded symmetrically along an axis, which causes uniformly distributed internal stress.

The Pirelli Office Block, designed by Ponti and engineered by Nervi, is a wonderful example of a structure in which the columns express not only the relationship between the moment and the building, but of the relationship between direct stress and size as well. The stress in the column was intended to remain as constant as possible.



PIRELLI OFFICE BLOCK. MILAN. PONTI.

EXAMPLE PROBLEM Stress/Area Relationship

EXAMPLE PROBLEM Compressive Stress

A **stress prism** is a diagram which represents the stress distribution across the section of a loaded member. The volume of the stress prism is equal to the total force acting on the cross-section.

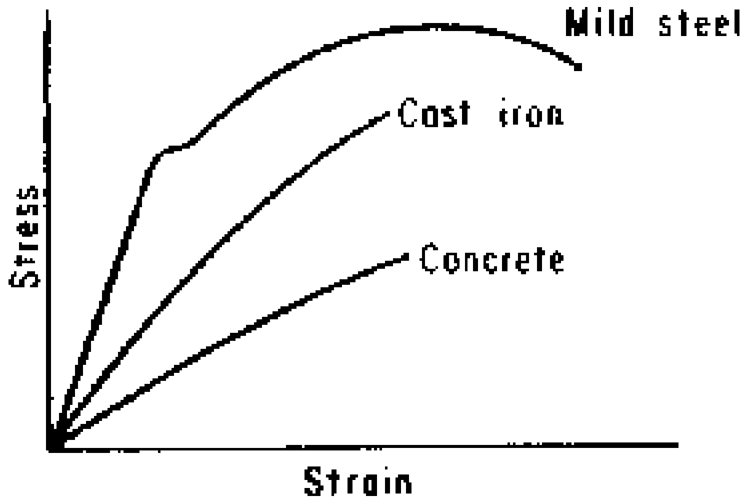
EXAMPLE PROBLEM Drawing a Stress Prism

EXAMPLE PROBLEM More Stress Prisms

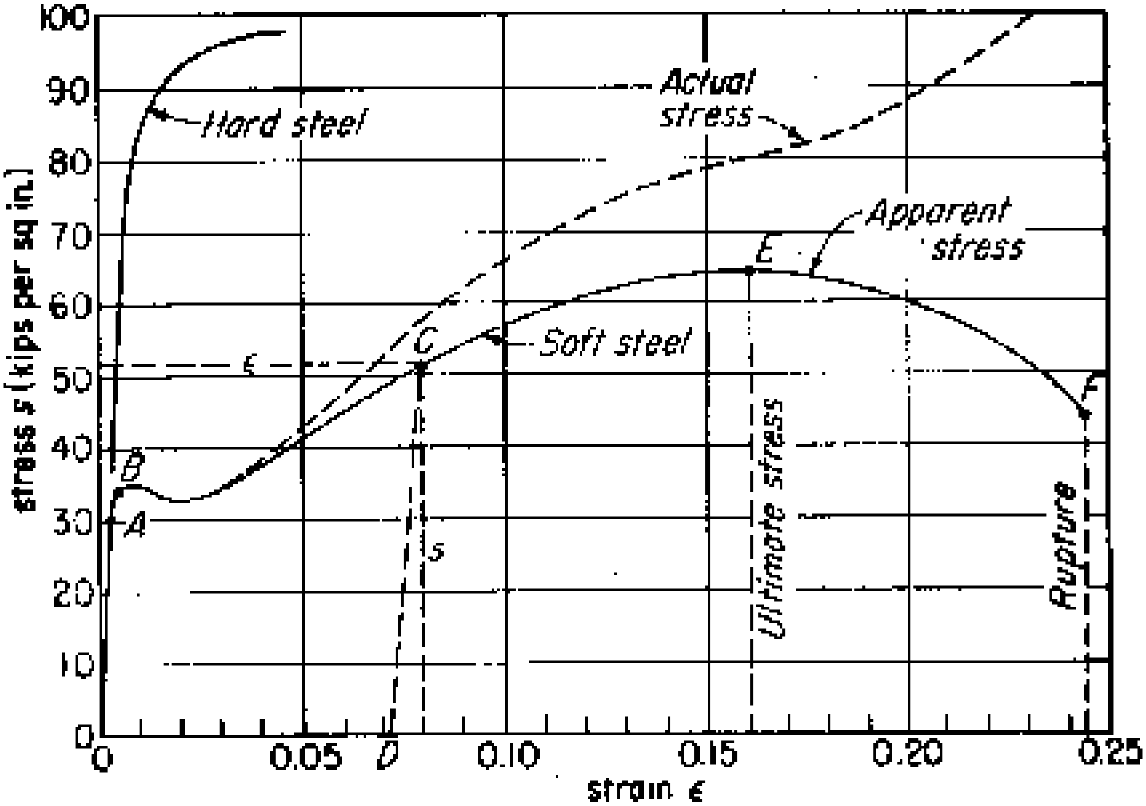
Lecture 24:

Stress-Strain Curves

The relationship between the stress and strain that a material displays is known as a **Stress-Strain curve**. It is unique for each material and is found by recording the amount of deformation (strain) at distinct intervals of tensile or compressive loading. These curves reveal many of the properties of a material (including data to establish the **Modulus of Elasticity, E**). What does a comparison of the curves for mild steel, cast iron and concrete illustrate about their respective properties?



It can be seen that the concrete curve is almost a straight line. There is an abrupt end to the curve. This, and the fact that it is a very steep line, indicate that it is a brittle material. The curve for cast iron has a slight curve to it. It is also a brittle material. Both of these materials will fail with little warning once their limits are surpassed. Notice that the curve for mild steel seems to have a long gently curving "tail". This indicates a behavior that is distinctly different than either concrete or cast iron. The graph shows that after a certain point mild steel will continue to strain (in the case of tension, to stretch) as the stress (the loading) remains more or less constant. The steel will actually stretch like taffy. This is a material property which indicates a high ductility. There are a number of significant points on a stress-strain curve that help one understand and predict the way every building material will behave.



An example plot of a test on two grades of steel is illustrated above. If one begins at the origin and follows the graph a number of points are indicated. Point A is known as the **proportional limit**. Up to this point the relationship between stress and strain is exactly proportional. The number which describes the relationship between the two is the Modulus of Elasticity. This is discussed in more detail in the next lecture.

Strain increases faster than stress at all points on the curve beyond point A. Up to this point, any steel speciment that is loaded and unloaded would return to its original length. This is known as elastic behavior. Point B is the point after which any continued stress results in permanent, or inelastic, deformation. Thus, point B is known as the **elastic limit**. Since the stress resistence of the material decreases after the peak of the curve, this is also known as the **yield point**.

The line between points C and D indicates the behavior of the steel specimen if it experienced continued loading to stress indicated as point C. Notice that the dashed line is parallel to the elastic zone of the curve (between the origin and point A). When the specimen is unloaded the magnitude of the inelastic deformation would be determined (in this case 0.0725 inches /inch). If the same specimen was to be loaded again, the stress-strain plot would climb back up the line from D to C and continue along the initial curve. Point E indicates the location of the value of the ultimate stress. Note that this is quite different from the yield stress. The yield stress and ultimate stress are the two values that are most often used to determine the allowable loads for building materials and should never be confused.

A material is considered to have completely failed once it reaches the ultimate stress. The point of rupture, or the actual tearing of the material, does not occur until point F. It is interesting to note the curve that indicates the actual stress experienced by the specimen. This curve is different from the apparent stress since the cross sectional area is actually decreasing. There is quite a bit to be learned from both the study of the ideal and actual behavior of all building materials. Changes in that body of knowledge have had large impacts on the way in which building structures are designed.

The earliest methods of design limited the stresses that a structure would be "allowed" to experience. Thus, the method of design was known as the Allowable Stress Method. Recognition of the additional strength potential of most materials resulted in the Ultimate Stress Method of design. Contemporary thought centers on the limitation of the various service conditions of the structure at hand. This is known as the Limit States Design method. In the end, it is the authors opinion that the actual method of design is less important than the legal bodies would like us to believe. Human factors in the construciton process SHOULD prevent a good designer from pushing too hard against the envelope of safety.

Additional Reading

Gere & Timoshenko. Mechanics of Materials, 3rd Edition. Chapter 1

COURSEWARE
STRUCTURES I LECTURES
BACK

Lecture 25:

Elasticity and Deformation

An important property of many structural materials is their ability to regain their original shape after a load is removed. These materials are called *elastic*. Steel, glass and rubber are elastic; putty or modeling clay are not elastic. Each of these materials is elastic to varying degrees; steel and glass are both more elastic than rubber. The degree of elasticity, or "stiffness" of a material is called its **Modulus of Elasticity (E)**. Given the modulus of elasticity, possible deformations can be calculated for any material and loading.

All materials are assumed to be elastic except when indicated otherwise in this course.

Robert Hooke (1635 - 1703), a great English scientist, experimented with springs, clocks and watches. During his investigation of the spring he discovered that in elastic materials, stress and strain are proportional. He first presented this in a lecture in 1678 and it is known today simply as **Hooke's Law**.

Hooke's Law applies as long as the material stress does not pass a certain point known as its proportional limit. This is the point at which the physical properties of the material actually change. Any time an elastic material is loaded between zero and the proportional limit, the stress and strain are directly proportional and if the load is released the material will regain its initial dimensions. If the stress is doubled the strain is doubled; if the stress is tripled the deformation is three times as great, etc.

The English physician and physicist Thomas Young (1773 - 1829) noted that if stress is proportional to strain, then for any given material, stress divided by strain would be a constant. This constant is known today as **Young's Modulus** or the **Modulus of Elasticity**.

The Modulus of Elasticity is represented by **E = Stress / Strain**.

This relationship is found as the slope of the curve of the stress-strain curve from initial loading to the proportional limit. A higher value of the modulus indicates a more brittle material (i.e. glass, ceramics). A very low value represents a ductile material (i.e. rubber).

EXAMPLE PROBLEM Modulus of Elasticity

The values of the modulus of elasticity for structural materials have been determined by tests and are readily available in references such as the AISC manual. Some of the more common values are:

- **Steel**: E=29,000 KSI (sometimes rounded to 30,000 KSI)
- **Aluminum**: E=10,000 KSI
- **Wood**: E=1,000 - 2,000 KSI (usual range)
- **Concrete** E=2,500 - 5,000 KSI (usual range)

It is often necessary to be able to determine the deformation of a structural member once the loads and physical properties of the structural member are known. This is simply derived and is developed from the stress/strain relationships that have already been established.

EXAMPLE PROBLEM Elastic Deformation

Additional Reading

Gere & Timoshenk. Mechanics of Materials, 3rd Edition. Chapter 1

COURSEWARE
STRUCTURES I LECTURES
BACK

Lecture 26:

Secondary Loads

Some types of loads on buildings are due to deformation. The deformation may be due to earth sliding or settling unevenly under the footings. This type of load can be avoided by properly designed footings on stable soil.

deformations due to earth settlement

The deformation of a building may be caused by temperature change. To avoid excessive stresses, construction must provide for a certain amount of thermal expansion and contraction. The problem is described in the diagram below.

forces due to thermal expansion and contraction

Lecture 27:

Thermal Stresses

The size of a body will change as the ambient temperature fluctuates, expanding as it rises and contracting as it falls. For example, cracks in an asphalt highway which were there all winter, dissapear during the summer. The asphalt closes the cracks as it expands in the summer heat but they will reappear in the cold of winter as the material shrinks. This repetative cycle of expansion and contraction results in significant temperature stresses in construction material.

Within the ordinary climatic temperature range experienced by buildings, the change in dimension has conveniently been found to be approximately proportional to the change in temperature. The amount of dimensional change that will take place is determined by a factor known as the **coefficient of thermal expansion (alpha)**. This describes the change in length (or width) of a member per unit length of the member. The values below describe some of the coefficients of thermal expansion for some common materials.

COEFFICIENTS OF THERMAL EXPANSION PER degree F

Wood	0.000 0030
Glass	0.000 0044
Concrete	0.000 0060
Cast Iron	0.000 0061
Steel	0.000 0065
Wrought Iron	0.000 0067
Copper	0.000 0093
Bronze	0.000 0100
Brass	0.000 0104
Aluminum	0.000 0128

These values describe the change in length per unit of length for one degree F temperature change. Some tables list the coefficients for a 100F temperature change. Similar tables may be found for the temperature change based on Centigrade rather than Fahrenheit. These are not only used to determine

the values for expansion, but for contraction or shortening as well.
The equation used in determining the deformation is as follows:

Delta L = (alpha)(delta T)(L)

Where:

delta L= total change in length, inches (total deformation)
alpha = coefficient of thermal expansion
delta T = temperature change
L = length in inches

Note that the equation does not include a variable for the cross-sectional area. Every construction material will expand and contract with changes in temperature; regardless of the cross-sectional area.

EXAMPLE PROBLEM Thermal Strain

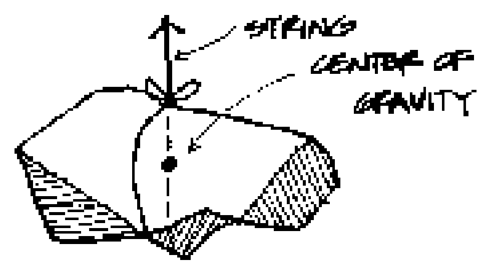
Temperature stresses can cause significant loads for high-rise and long-span structures. In both cases the initial design and construction detailing must allow for these dimensional changes to prevent excessive stresses and strains.

Lecture 28:

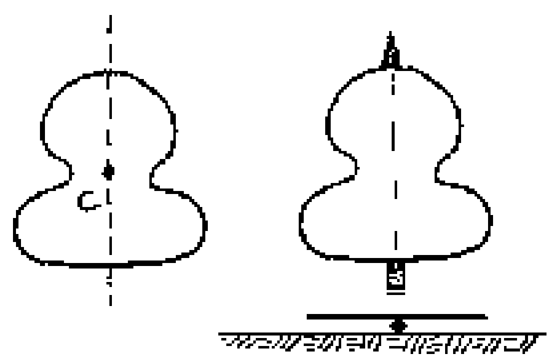
Centroids & Moment of Inertia

The **centroid**, or center of gravity, of any object is the point within that object from which the force of gravity appears to act. An object will remain at rest if it is balanced on any point along a vertical line passing through its center of gravity. In terms of moments, the center of gravity of any object is the point around which the moments of the gravitational forces completely cancel one another.

The center of gravity of a rock (or any other three dimensional object) can be found by hanging it from a string. The line of action of the string will always pass through the center of gravity of the rock. The precise location of the center of gravity could be determined if one would tie the string around the rock a number of times and note each time the line of action of the string. Since a rock is a three dimensional object, the point of intersection would most likely lie somewhere within the rock and out of sight.

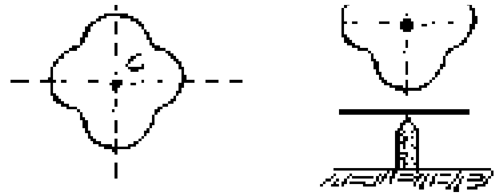


The centroid of a two dimensional surface (such as the cross-section of a structural shape) is a point that corresponds to the center of gravity of a very thin homogeneous plate of the same area and shape. The planar surface (or figure) may represent an actual area (like a tributary floor area or the cross-section of a beam) or a figurative diagram (like a load or a bending moment diagram). It is often useful for the centroid of the area to be determined in either case.

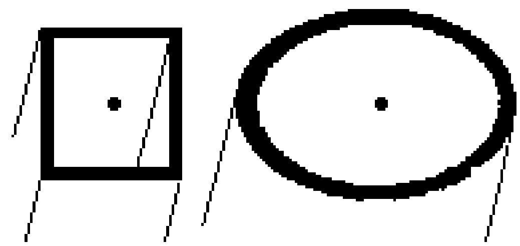


Symmetry can be very useful to help determine the location of the centroid of an area. If the area (or section or body) has one line of symmetry, the centroid will lie somewhere along the line of symmetry. This means that if it were required to balance the area (or body or section) in a horizontal position by placing a pencil or edge underneath it, the pencil would be best laid directly under the line of symmetry.

If a body (or area or section) has two (or more) lines of symmetry, the centroid must lie somewhere along each of the



lines. Thus, the centroid is at the point where the lines intersect. This means that if it were required to balance the area (or body or section) in a horizontal position by placing a nail underneath it, the point of the nail would best be placed directly below the point where the lines of symmetry meet. This might seem obvious, but the concept of the centroid is very important to understand both graphically and numerically. The position of the center of gravity for some simple shapes is easily determined by inspection. One knows that the centroid of a circle is at its center and that of a square is at the intersection of two lines drawn connecting the midpoints of the parallel sides. The circle has an infinite number of lines of symmetry and the square has four. (Two were described above - what are the other two lines of symmetry?)



The centroid of a section is not always within the area or material of the section. Hollow pipes, L shaped and some irregular shaped sections all have thir centroid located outside of the material of the section. This is not a problem since the centroid is really only used as a reference point from which one measures distances. The exact location of the centroid can be determined as described above, with graphic statics, or numerically.

The centroid of any area can be found by taking moments of identifiable areas (such as rectangles or triangles) about any axis. This is done in the same way that the center of gravity can be found by taking moments of weights. The moment of an large area about any axis is equal to the algebraic sum of the moments of its component areas. This is expressed by the following equation:

$$\textbf{Sum } M_{A_{total}} = M_{A_1} + M_{A_2} + M_{A_3} + \ldots$$

The moment of any area is defined as the product of the area and the perpendicular distance from the centroid of the area to the moment axis. By means of this principle, we may locate the centroid of any simple or composite area.

EXAMPLE PROBLEM Center of Gravity

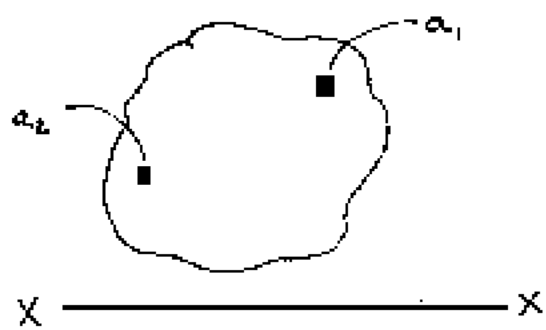
The **Moment of Inertia (I)** is a term used to describe the capacity of a cross-section to resist bending. It is always considered with respect to a reference axis such as X-X or Y-Y. It is a mathematical property of a section concerned with a surface area and how that area is distributed about the reference axis. The reference axis is usually a centroidal axis.

The moment of inertia is also known as the **Second Moment of the Area** and is expressed mathematically as:

$$\mathbf{I_{xx} = Sum\ (A)(y^2)}$$

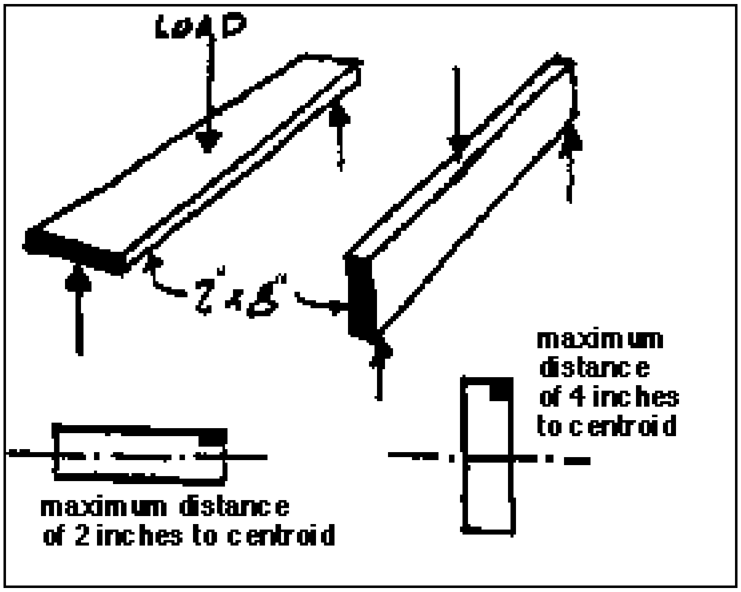
In which:

I_{xx} = the moment of inertia around the x axis
A = the area of the plane of the object
y = the distance between the centroid of the object and the x axis



The Moment of Inertia is an important value which is used to determine the state of stress in a section, to calculate the resistance to buckling, and to determine the amount of deflection in a beam. For example, if a designer is given a certain set of constraints on a structural problem (i.e. loads, spans and end conditions) a "required" value of the moment of inertia can be determined. Then, any structural element which has at least that specific moment of inertia will be able to be utilized in the design. Another example could be in the inverse were true: a specific element is given in a design. Then the load bearing capacity of the element could be determined.

Let us look at two boards to intuitively determine which will deflect more and why. If two boards with actual dimensions of 2 inches by 8 inches were laid side by side - one on the two inch side and the other on the eight inch side, the board which is supported on its 2" edge is considerably stiffer than that supported along its 8" edge. Both boards have the same cross-sectional area, but the area is distributed differently about the horizontal centroidal axis.



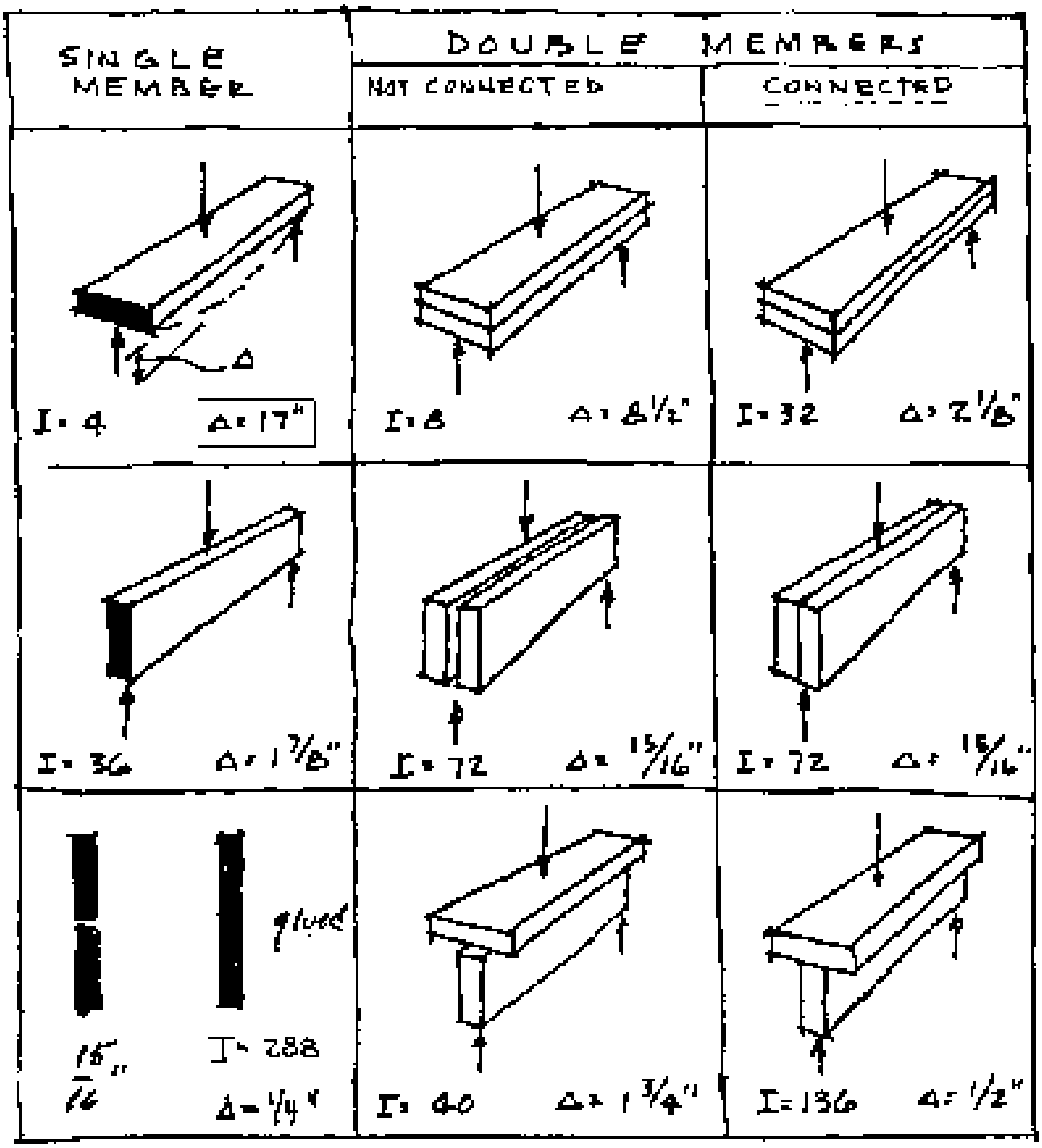
Calculus is ordinarily used to find the moment of inertia of an irregular section. However, a simple formula has been derived for a rectangular section which will be the most important section for this course.

$$\mathbf{I_{xx} = (^{1}/_{12}) (b)(h^{3})}$$

In which the value **b** is **alway**s taken to be the side parallel to the reference axis and **h** the height of the section. This is very important to note! If the wrong value is assumed for the value of **b**, the calculations will be totally wrong.

EXAMPLE PROBLEM   Moment of Inertia

The importance of the distribution of the area around its centroidal axis becomes clear when comparing the values of the moment of inertia of a number of typical beam configurations. All of the members shown below are 2" x 6"; in cross section, equal in length and equally loaded.



BUILT-UP SECTIONS

It is often advantageous to combine a number of smaller members in order to create a beam or column of greater strength. The moment of inertia of such a built-up section is found by adding the moments of inertia of the component parts. This can be done< B> if and only if the moments of inertia of each component area are taken about a common axis and **if and only if** the resulting section acts as one unit.

UNDER NO OTHER CONDITION CAN THEY BE ADDED!

Two examples of built-up sections are seen below. In each case the components of the whole have a common axis and act as one unit.

EXAMPLE PROBLEM Built-Up Sections

TRANSFER FORMULA

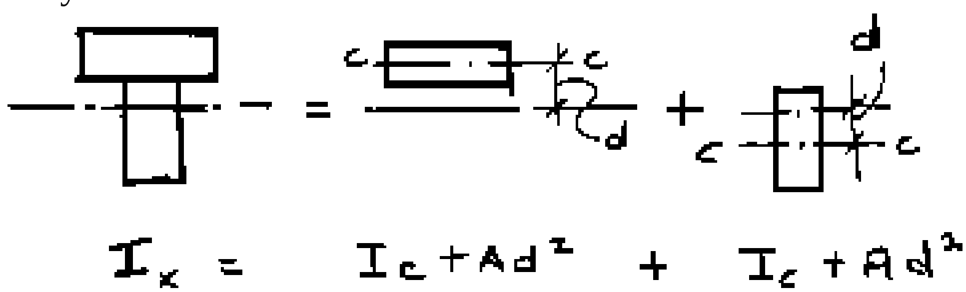
There are many built-up sections in which the component parts are not symmetrically distributed about the centroidal axis. The easiest way to determine the moment of inertia of such a section is to find the moment of inertia of the component parts about their own centroidal axis and then apply the transfer formula. The **transfer formula** transfers the moment of inertia of a section or area from its own centroidal axis to another parallel axis. It is known from calculus to be:

$$\mathbf{I_x = I_c + (A)d^2}$$

Where:

> $\mathbf{I_x}$= moment of inertia about axis x-x (in^4)
>
> $\mathbf{I_c}$= moment of inertia about the centroidal axis c-c parallel to x-x (in^4)
>
> **A**= area of the section (in^2)
> **d**= perpendicular distance between the parallel axes x-x and c-c (in)

Finding the moment of inertia of an asymmetric built-up cross-section is simplified to the procedure shown diagrammatically below:

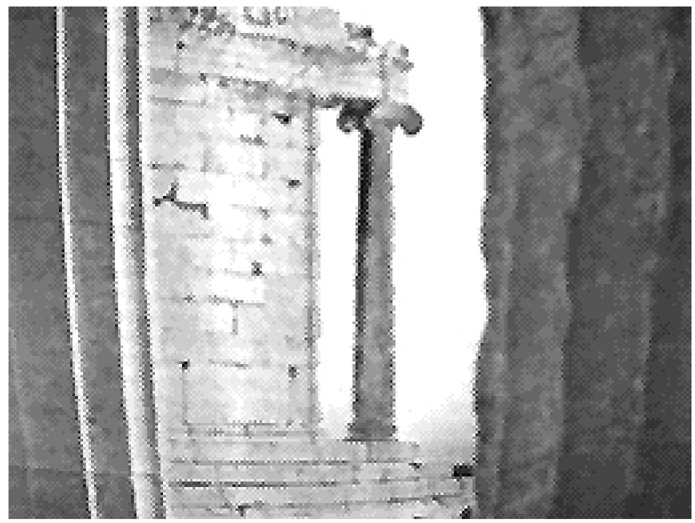


EXAMPLE PROBLEM [Asymmetric Cross Sections]

EXAMPLE PROBLEM [Bridge Girder Section]

Lecture 29:

Columns



The column is one of the most familiar architectonic icons. Each of us can picture this very important space definer in our mind's eye. It can delineate, punctuate, or seperate spaces. A quick glance out most windows allows a view of at least one of them. Columns are vertical load-bearing elements which are normally loaded in compression. Time has brought many manifestations of this simple element; thick, thin, long, short, spindle shaped, squat, etc. And they have been made out of any material that has a minimal compressive strength.

The transition between the column and its loading member, usually a beam of some sort, has often been of great interest to builders. The structural/functional problem is quite simple - the forces collected by the beam must be transferred from the beam to the column - and yet the solutions demonstrated a great variation. The early Egyption, Greek and Roman civilizations turned this transition zone into an articulated flourish to draw attention to the capital. In almost every case it was broader at the top and tapered to the column shaft. This not only facilitated a smooth transition for the forces, but also eased the construction and even increased the stability of of the buildings.

A column either crushes (a strength falure) or it buckles (a stability failure). Both modes of failure must be considered for every column. The exact mode of failure is greatly dependent upon the way in which the column's cross-sectional area is distributed with respect to its centroid. The following simple concept must be satisfied at all times:

Stress due to loading < Resistance potential of the column

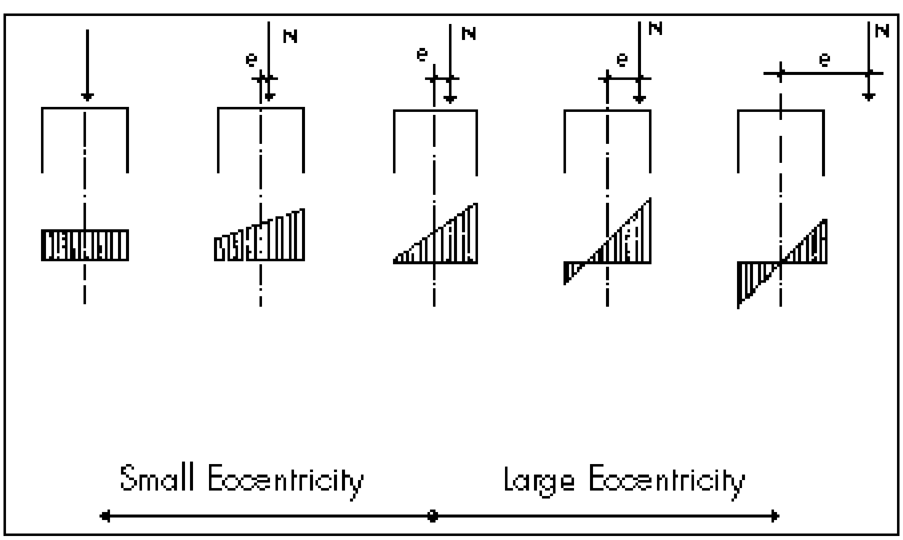
This states that the stress within the column due to all of the applied loads must be less than the allowable stress of the material. This is a logical statement that is the essence of the structural analysis of a column. The actual failure mechanism could be due to a combination of two or more loads. These combinations must be carefully considered.

Column Failure Modes

CRUSHING
Relatively short columns are more apt to fail by the material crushing. Every building material material can withstand a distinct amount of compressive stress before it crushes. This value has been determined by laboratory tests and is known as the compressive strength of a material. This strength is dependent upon the internal structure of the material and/or its components. Steel has a very homogeneous, finely crystalled internal structure and has a relatively high compressive strength. Hard woods are fine-grained and have a higher compressive strength than soft-woods. Wood is unique in that it has two compressive strengths; one when loaded parallel to the grain and another when loaded perpendicular to the grain. Why is that? When a wood column crushes the fibers of the wood actually split apart. In every case, crushing is a strength failure and does not depend upon the shape of the section.

BUCKLING
Relatively slender columns are more apt to fail by buckling. A column is slender when it has a "small" cross-section compared to its effective length. Small is placed in quotes due to the fact that the important information about the cross section is both the actual size and more importantly, the shape of the cross-section. This is then compared to the effective length to determine whether or not the column is slender. If it is, this means that the column will probably fail in bending! As a column is loaded, it is likely to bend about the weak axis of the cross-section (the one with the lowest Moment of Inertia). A column **buckles** when it bends about an axis. This is a stability failure.

TYPES OF LOADING
There are generally two types of column loading: axial and eccentric. Axially loaded columns may fail either by crushing or buckling. Eccentrically loaded columns usually fail by buckling. The figure below illustrates the stress that a column experiences as a load, **N**, is applied with increasing eccentricity. Note how the form of the stress prism changes from an even distribution to a very uneven distribution.



The case at the left illustrates how the axial load creates a compressive stress which is evenly distributed across the column's section. The load on each column to the right has an increasing eccentricity. As the load moves away from the centroidal axis, it introduces a bending moment which the column's cross-section must also resist. Thus, one can see that one side of the column receives more compression than the other. As long as the applied load remains within what is known as the core (the middle third) of the section, the column cross-section will only have compressive stresses. Tesion stresses are introduced as soon as the applied load moves out of the core. The magnitude of the bending moment that the section must also resist increases as the eccentricity increases. The extreme case is an infinite eccentricity resulting in the pure bending moment stress prism (like that found in a beam) that is seen in the case furthest to the right.

Knowledge of the magnitude and distribution of the internal stress is important for the sizing of the column. A small amount of tensile stress has little effect on a wood or steel column, but problems could begin to occur if the column is concrete or masonry.

Lecture 30:

Radius of Gyration & Buckling

The **radius of gyration (r)** describes the way in which the area of a cross-section is distributed around its centroidal axis. If the area is concentrated far from the centroidal axis it will have a greater value of r and a greater resistance to buckling. A cross-section can have more than one radius of gyration and most sections have at least two. If this is the case, the section tends to buckle around the axis with the smallest value. The radius of gyration is defined as:

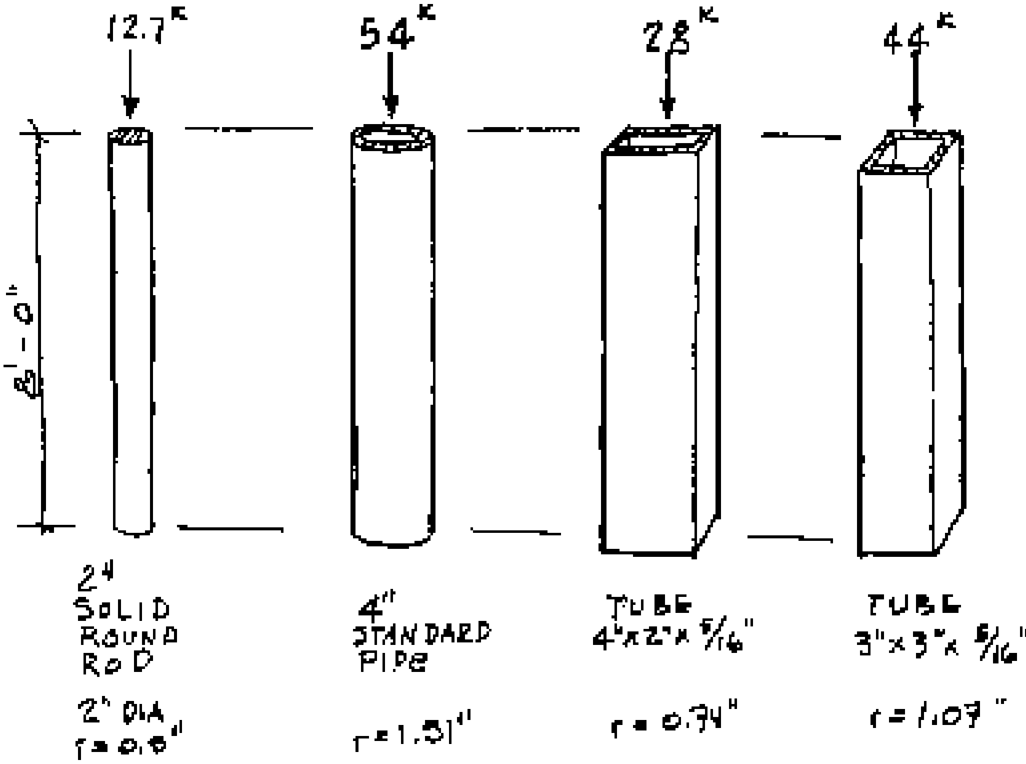
$$\mathbf{r = sqrt\ (I/A)}$$

where

r = radius of gyration
I = moment of inertia
A = area of the cross section

All things being equal, a circular pipe is the most efficient column section to resist buckling. This is because it has an equal radius of gyration in all directions and it has the its area distributed as far away as possible from the centroid.

The steel columns shown below all have areas of 3-1/8 in^2. The safe loads for an 8 ft length are shown. The only difference between them is the way in which the cross-sectional area is distributed about the centroid.



Buckling

Buckling is very similar to bending. Thus, the shape of the cross-section is very important. The shape of the column also effects the way in which it will buckle. Imagine for a moment a single sheet of paper (A4 or 8.5 x 11). If one would try to simply stand it on edge it would be impossible unless the paper was folded. This simple act of folding the paper actually increases the cross-sectional moment of Inertia and thus the stiffness of the newly formed column. The stiffness of another paper column could be futher increased by taking the paper and taping the long edges together to create a tube. Now, the paper will be very stiff since the material of the paper is distributed evenly as far away from the neutral axis as possible.

The load at which a column will begin to buckle is known as the Critical Buckling Load (or critical load). A number of qualities of the column must be known in order to determine this value. The Swiss mathematician Leonard Euler (1707 - 1783) derived a formula in 1744 (known as the Euler Buckling Formula) to determine the load at which a perfect column will buckle. It was a very important step in the history of technology and remains important for column design today. The equation is only accurate for columns which approach the perfect conditions for which he derrived the equation.

$$N_{cr} = ¼^2 E\ I / (l_k^2)$$

In which the terms are defined as follows:

N_{cr} = Critical Buckling Load

E = Modulus of Elasticity
I = Moment of Inertia
l_k = Effective Buckling Length

This equation can be modified by dividing both sides by the area of the column so that the stress at which the column will buckle can be determined:

f_{cr} = sigma$_{cr}$ = N_{cr} / A

= ¼^2 E I /A(l_k^2)

now, knowing that r^2 = I/A, this equation becomes:

= ¼ ^{2} E /(l_k/r)^2

Lambda, or the **slenderness ratio** is a value with which one can gage the relative resistance of a column cross-section to buckling. Or, stated otherwise, the relative ease in which a column WILL buckle. It is defined as

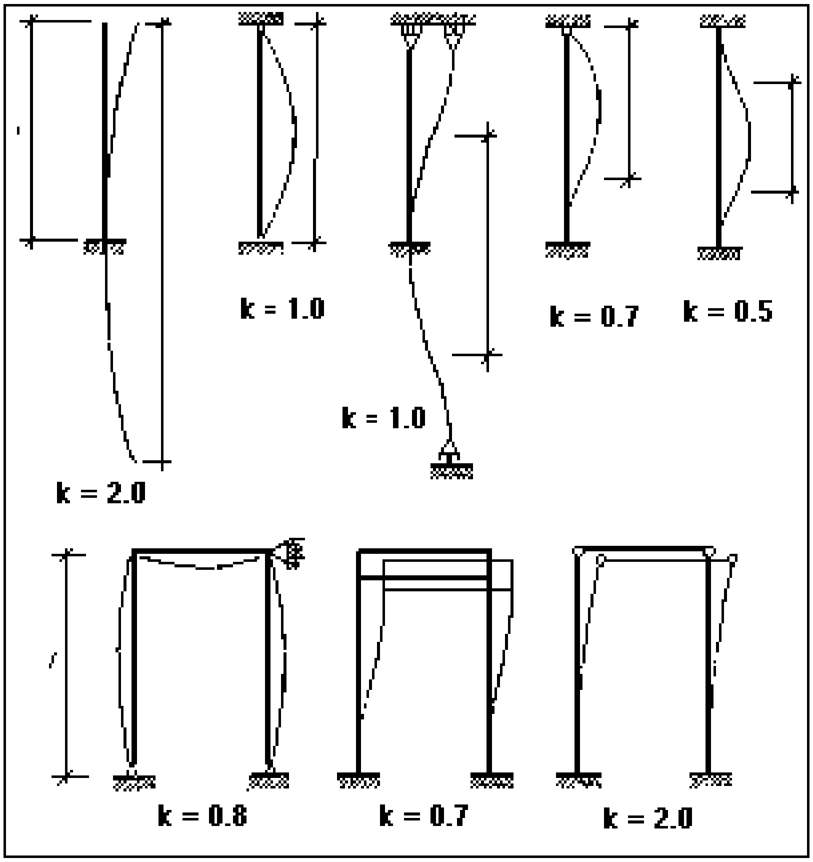
lambda = l_k / r

Where l_k is the buckling length and r is the radius of gyration.

Thus, the critical buckling stress can be expressed as,

f_{cr}= ¼^2 E / (lambda)^2

The **buckling length** of a column depends on its physical length and its end conditions. Euler discovered that if a column is hinged at both ends it will buckle in the form of a sine curve with the inflection points at the hinges. This would be the case in which the buckling length of a column is identical to its length. This is not the case if the ends of the column are both fixed. The determination of the buckling lengths for various column end conditions and frames is given below:



The magnitude of the internal forces is important to know in order to size a column. A small amount of tensile stress has little effect on a wood or steel column, but could cause problems in a concrete or masonry column.

Questions for Thought

What are the relationships between the various columnar elements within the human skeleton? What are the various end conditions? What is the buckling length of the columns in a fassade near you?

Additional Reading

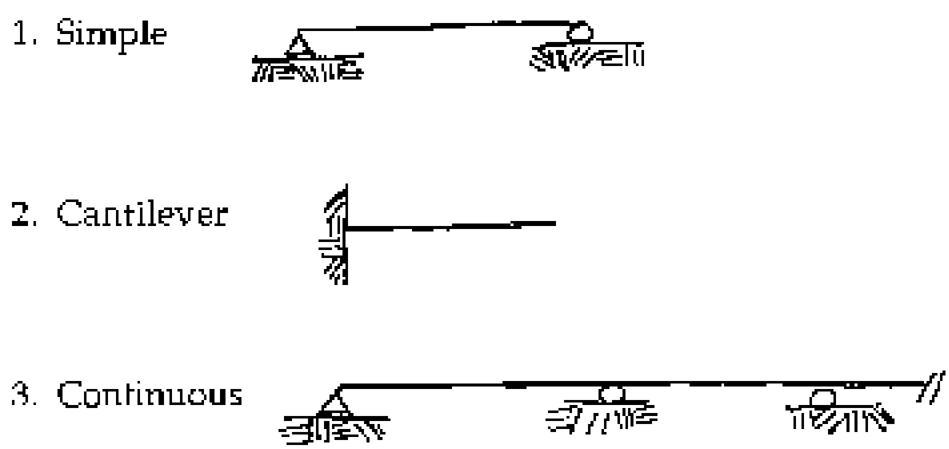
Schodeck, Daniel. "Structures." Chapter 7.

Lecture 31:

Beams

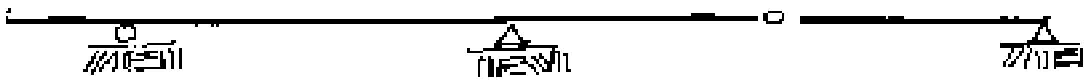
A **beam** is a structural member which carries loads. These loads are most often perpendicular to its longitudinal axis, but they can be of any geometry. A beam supporting any load develops internal stresses to resist applied loads. These internal stresses are bending stresses, shearing stresses, and normal stresses.

Beam types are determined by method of support, not by method of loading. Below are three types of beams that will be investigated in this course:

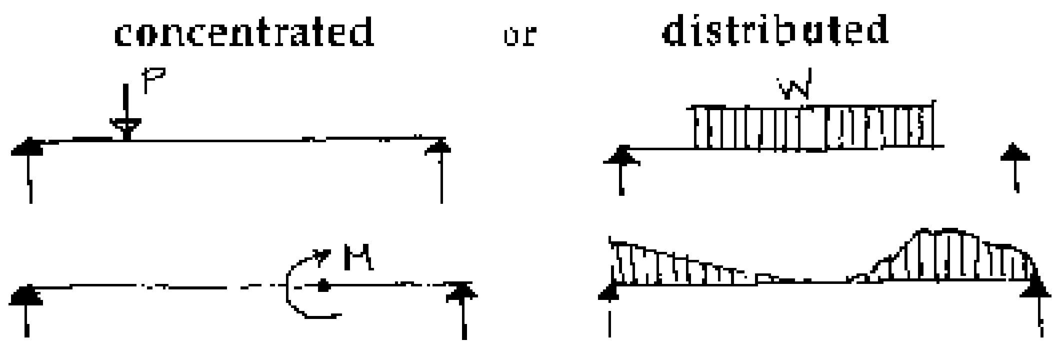


The first two types are statically determinate, meaning that the reactions, shears and moments can be found by the laws of statics alone. Continuous beams are statically indeterminate. The internal forces of these beams cannot be found using the laws of statics alone. Early structures were designed to be statically determinate because simple analytical methods for the accurate structural analysis of indeterminate structures were not developed until the first part of this century. A number of formulas have been derived to simplify analysis of indeterminate beams.

The three basic beam types can be combined to create larger beam systems. These complex systems can *inevitably* be distilled to the simple beam types for analysis. The beams shown immediately below are combinations of the first two beam types; these systems are all statically determinate.



The two beam loading conditions that either occur separately, or in some combination, are:



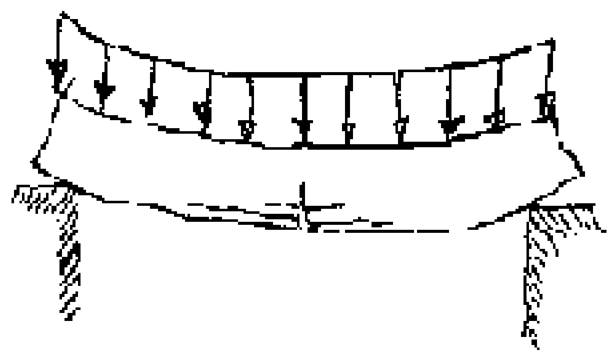
CONCENTRATED
Either a force or a moment can be applied as a concentrated load. Both are applied at a single point along the axis of a beam. These loads are shown as a "jump" in the shear or moment diagrams. The point of application for such a load is indicated in the diagram above. Note that this is NOT a hinge! It is a point of application. This could be point at which a railing is attached to a bridge, or a lampost on the same.

DISTRIBUTED
Distributed loads can be uniformly or non-uniformly distributed. Both types are commonly found on all kinds of structures. Distributed loads are shown as an angle or curve in the shear or moment diagram. A uniformly distributed load can evolve into a n on-uniformly distributed load (snow melting to ice at the edge of a roof), but are normally assumed to act as given. These loads are often replaced by a singular resultant force in order to simplify the structural analysis.

Lecture 32:

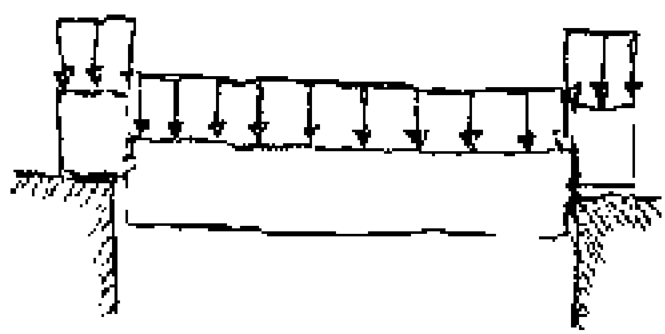
Beam Failure Modes

Structures fail in many ways; two categories of which have been described: stability failures and strength failures. Stability failures usually relate to structural systems, whereas strength failures relate to the members comprising a structure. Of the many ways which a beam can fail, three will be discussed in this course: bending, vertical shear, and horizontal shear.

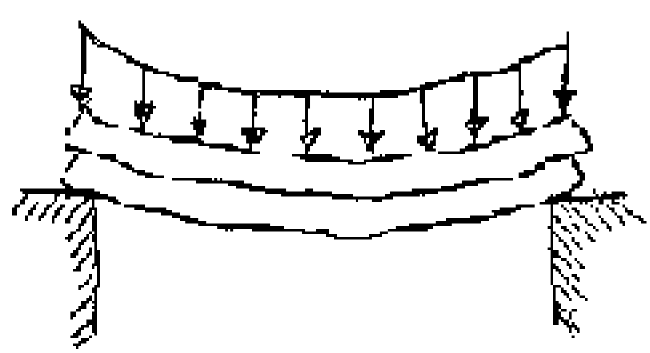


Bending is probably the most common type of failure. It is illustrated by the top figure in which the "fibers" along the bottom face of the beam are torn and those along the top face of the beam are crushed.

Vertical Shear is an idealized mode of failure. There is a tendency for a short beam to fail in this manner. This is the way in which a pair of "shears" or scissors cuts a piece of paper.



Horizontal Shear is the tendancy for a material to separate parallel to the neutral axis as its "internal layers" try to slide past each other. It is a frequent mode of failure which should not be confused with checking in wood beams.



Our discussion during the next few days will be limited to the stresses that are related to these modes of failure. Bending and horizontal shear are the types of failure most often encountered. Vertical shear is useful to us primarily because of what it reveals about horizontal shear. Excessive deflection can also be a cause of failure.

Peting
16V95CHL

COURSEWARE
STRUCTURES I LECTURES
BACK

Lecture 33:

Internal Forces

Internal forces are generated within loaded structural elements. These forces are generated within every type of element; if they were not developed, the structure would fail. These are known as **Shear**, **Moment**, and **Normal Forces** . The normal force is found in columns and beams with an axial load. Shear and moment are found in beams and frames. Most of the elements that will be analyzed in this course will be beams, including joists, purlins, girders, decking, planking, etc.

Shear and moment are essential for developing an understanding of how structures behave. Considerable time will be spent on this subject because of its importance now and in the future. The capacity to draw shear and moment diagrams for any designed structure will add to the repertoire of the designer. All of the great Modern architects had this basic skill.

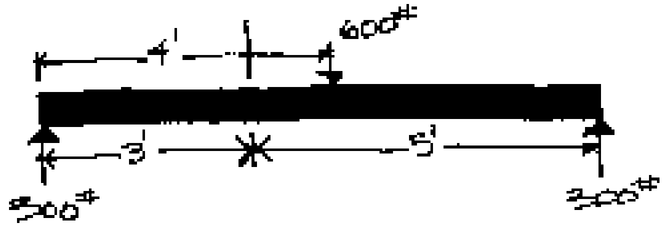
In order to quantify the internal forces of any part of a beam a FBD must be drawn for *only* that specific part of the beam. If the entire structure is in equilibrium, this part, when isolated from the remainder as a Free Body, must also be in equilibrium. The action of the external forces on that part must be resisted by the internal forces acting at the cut section(s).

EXAMPLE PROBLEM   Internal Forces

External Moment = Internal Moment
or
External Moment - Internal Moment = 0

Internal resisting forces and moments caused by the loading on a beam usually change along the length of the beam. Let us again examine the internal forces of a weightless beam by means of a FBD:

A beam carrying a load is clearly in equilibrium.



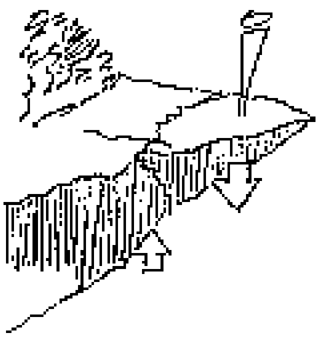
If we cut the beam and apply the forces on the cut section which existed internally before the cut, the beam will still be in equilibrium. We can find these forces by applying equations of equilibrium on the ends of the cut sections (as in any FBDs).

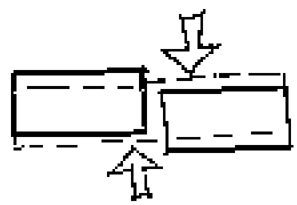
The solutions to the equations of equilibrium are the internal normal (N), shear (V) and moment (M) forces at that section. This is then applied to each and every point along the beam.

Lecture 34:

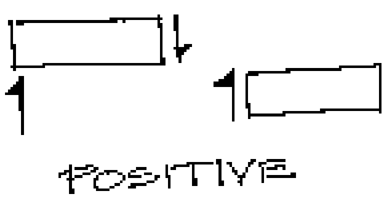
Shear (V) and Shear Diagrams

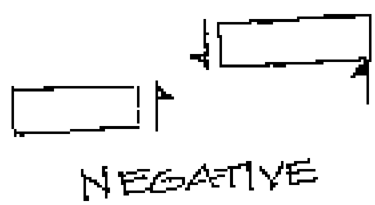
Shear (V) is the tendency for one part of a beam to slide past another part. The magnitude of the shear at any section is equal to the algebraic sum of loads and reactions acting perpendicular to that section.





SIGN CONVENTION
If the tendency of the section to the left of the cut is to move upward, the shear is positive; if it has a tendency to move down, it is negative.





For simplicity in obtaining the correct sign, one may say that it is equal to the algebraic sum of loads and reactions to the left of the cut. Note that the internal shear on the end of the FBD acts in the opposite direction of the algebraic sum of the loads and reactions in order to balance these forces (SV = 0).

A **shear diagram** is a graphic representation of the shear at every point along the length of a member.

To plot a shear diagram on a beam, the shear must be calculated at each point along the length of the beam. One way is to simply plot the shear as the algebraic sum of the loads and reactions acting perpendicular to the beam at the left side of each increment along the length of the beam. Positive values are shown above and negative values below a reference axis.

How to draw a SHEAR DIAGRAM
Start at the left end and plot the external shear values with regard to the following:

- The shear diagram is the graphic representation of the shear force at successive points along the beam. Forces acting upward are assumed positive and downward forces negative.
- The shear force (V) at any point is equal to the algebraic sum of the external loads and reactions, perpendicular to the beam, to the left of that point.
- Since the entire beam must be in equilibrium (sum of V = 0), the shear diagram must close to zero at the right end.
- Consider the loading for increments along the length of the beam in order to determine the shape of the curve.

> if there is no change in the load along the incremental length under consideration, the shear curve is a straight horizontal line (or a curve of zero slope). The slope at any point is defined as the tangent to the curve at that point.
>
> if a load exists, but does not change in magnitude over successive increments (uniformly distributed), the slope of the shear curve is constant and non-horizontal.
>
> if a load exists, and increases in magnitude over successive increments, the slope of the shear curve is positive (approaches the vertical); if the magnitude decreases, the slope of the shear curve is negative (approaches the horizontal).

- Abrupt changes in loading cause abrupt changes in the slope of the shear curve. Concentrated loads produce vertical lines (a jump) in the shear curve.

EXAMPLE PROBLEM Shear Diagrams

Lecture 35:

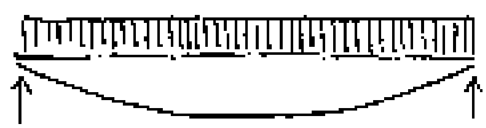
Moment (M) and Moment Diagrams

The **Moment (M)** within a beam is a representation of the magnitude of the internal couple found within the beam at any given point.

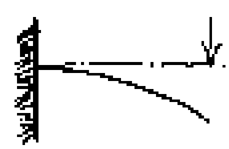
A **moment diagram** is a graphic representation of the moment at every point along the length of the member. To plot a moment diagram of a beam, one must determine the magnitude of the moment at each point along the length of the beam and then determine whether it is a positive or negative moment. Positive values are shown above and negative values below a reference axis.

Sign convention is probably the most troublesome part of moment diagrams. In North America, if the moment tends to cause the beam to **curve upward** it is **positive**; if the moment tends to cause it to **curve downward** it is **negative**. This is the opposite of the conventions for most of the rest of the world.

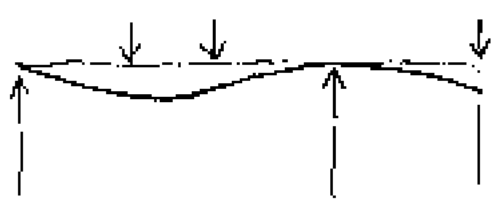
The sign of the moment diagram is not necessarily the same sign of the moment found in the FBD or in the moment equation when determining its magnitude.



A simplly supported beam with gravity load always has a *positive* moment.



A cantilever with gravity load always has a *negative* moment.



A continuous beam with an overhang subject to a gravity load experiences both *positive and negative* moments.

How to draw a MOMENT DIAGRAM

Start at the left end and plot the bending moment values due to external loads and reactions with regard to the following:

- The moment diagram is the graphic representation of the magnitude of the bending moment at successive points along the beam.

- The bending moment for the moment diagram (M) at any point equals the sum of moments of the forces to the left about that point.
- Since the entire beam is in equilibrium (sum of M=0), the bending moment diagram must close to zero at the right side.

- Consider the values of successive incremental ordinates along the length of the beam in order to determine the shape of the curve:

  if the magnitudes of successive shear ordinates are constant, the moment curve has a constant slope at that increment.

  if the magnitudes of successive shear ordinates increase, the slope of the moment curve is positive (it approaches the vertical).

  if the magnitudes of successive shear ordinates decreases, the slope of the moment curve is negative (it approaches the horizontal).

- Abrupt changes in the shear diagram will produce abrupt changes in the slope of the moment curve. Concentrated moments produce vertical lines in the moment curve.

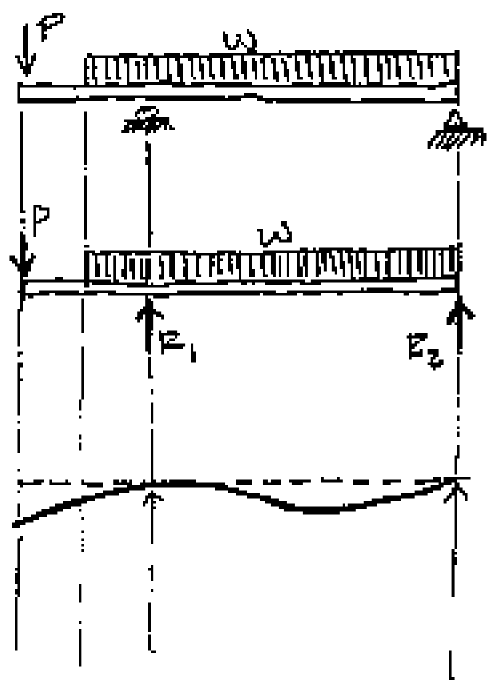
EXAMPLE PROBLEM Moment Diagrams

COURSEWARE
STRUCTURES I LECTURES
BACK

Lecture 36:

Plotting V, M and Deflection

A comprehensive procedure for the construction of shear, moment, and deflection diagrams for statically determinate beams is given below. Each and every one of these diagrams are plotted from **left to the right** and positive values are shown above, and negative values below, a reference axis. The procedure is as follows:



1. Draw a FBD of the entire structure and find the reactions of the beam.

2. Based on intuition and an educated guess, sketch an **approximate deflection curve** of the structure. this will help to determine the location of inflection points and thus help to construct the diagrams.

3. Draw a **SHEAR DIAGRAM**. Start at the left end and plot the external shear values with regard to the following:

- The shear diagram is the graphic representation of the shear force at successive points along the beam. Upward acting forces are assumed positive and downward forces negative.

- The shear force (V) at any point is equal to the algebraic sum of the external loads and reactions to the left of that point.
- Since the entire beam must be in equilibrium (the sum of V = 0), the shear diagram **must** close to zero at the right end.
- Consider the loading for increments along the beam's length in order to determine the shape of the curve.

    if there is no change in the load along the incremental length under consideration, the shear curve is a straight horizontal line (or a curve of zero slope). The slope at any point is defined as the tangent to the curve at that point.

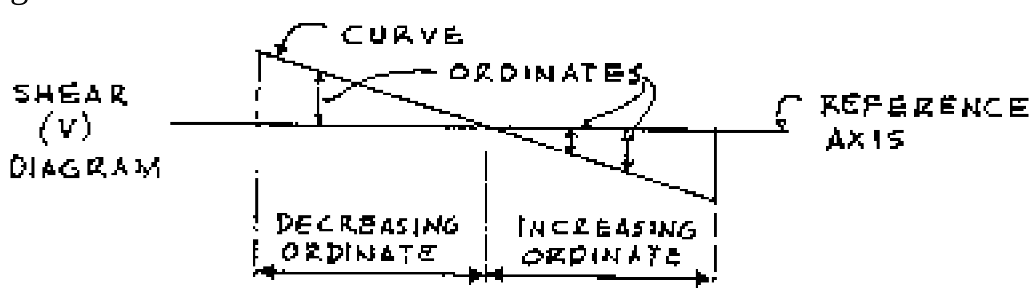
    if a load exists, but does not change in magnitude over successive increments (uniformly distrib-uted), the slope of the shear curve is constant and non-horizontal.

    if a load exists, and increases in magnitude over successive increments, the slope of the shear curve is positive (approaches the vertical); if the magnitude decreases, the slope of the shear curve is negative (approaches the horizontal).

- Abrupt changes in loading cause abrupt changes in the slope of the shear curve. Concentrated loads produce vertical lines (a jump) in the shear curve.

4. Draw a **MOMENT DIAGRAM**. Start at the left end and plot the bending moment values due to the external loads and the reactions with regard to the following:

- The moment diagram is the graphical representation of the magnitude of the bending moment at successive points along the beam.
- The bending moment for the moment diagram (M) at any point equals the sum of moments of the forces to the left about that point.
- Since the entire beam is in equilibrium (Sum of M=0), the bending moment diagram must close to zero at the right side.



- Consider the values of successive incremental ordinates along the length of the beam in order to determine the shape of the curve:

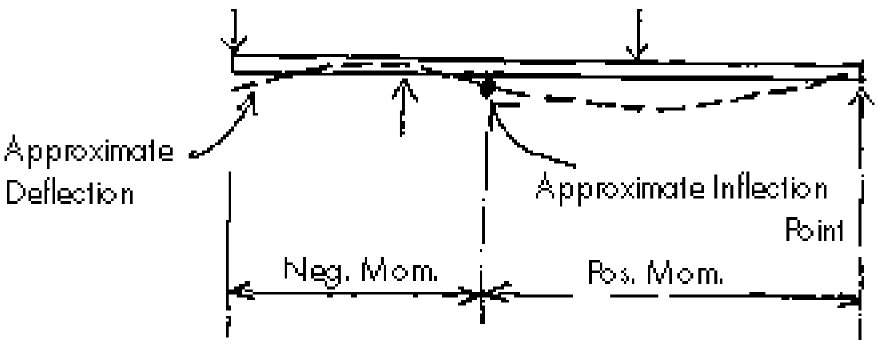
if the magnitudes of successive shear ordinates are constant, the moment curve has a constant slope at that increment.

if the magnitudes of successive shear ordinates increase, the slope of the moment curve is positive (it approaches the vertical).

if the magnitudes of successive shear ordinates decreases, the slope of the moment curve is negative (it approaches the horizontal).

- Abrupt changes in the shear diagram will produce changes in the shape of the moment curve. Concentrated moments produce vertical lines in the moment curve.

5. Draw a **DEFLECTION DIAGRAM**.



There is no set manner in which the deflection diagram should be drawn. There are, however, a few rules which can be followed:

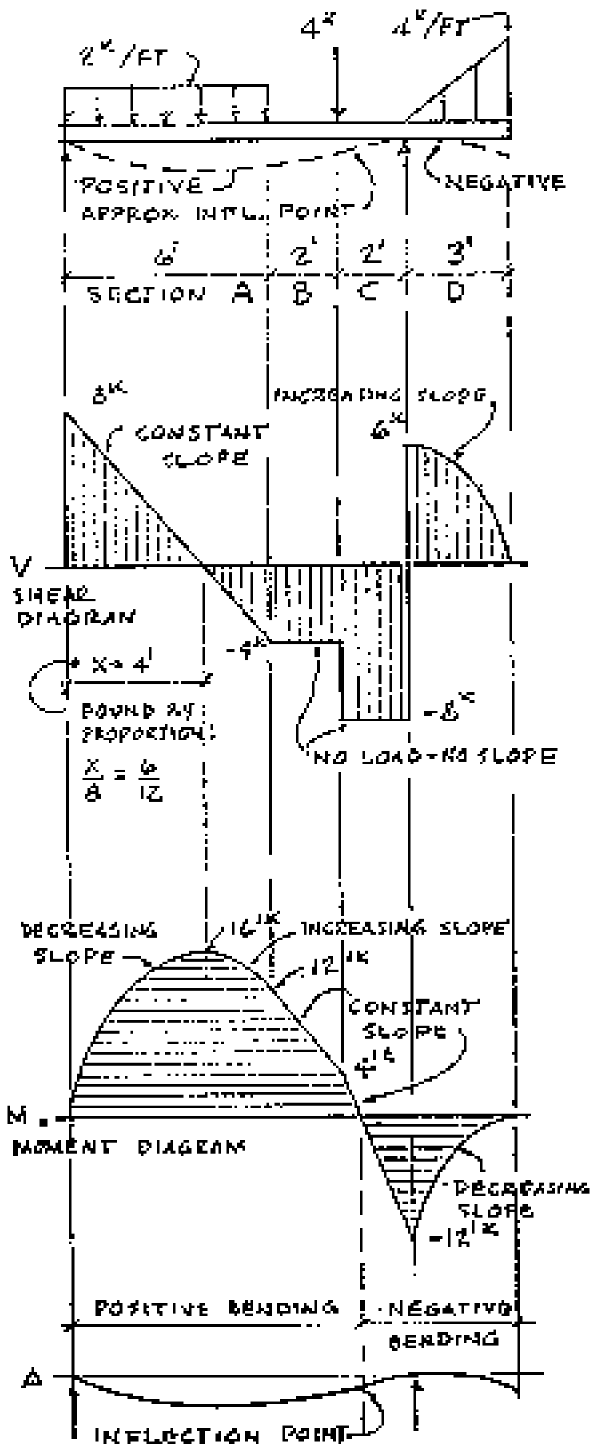
- points of zero moment are inflection points (change in curvature direction) of the deflection curve.
- the maximum points of the moment diagram are the points at which the deflection diagram slope is zero (where the tangent of the curve is horizontal).

6. There are a few relationships (based on calculus) between these three diagrams that are helpful to construct them:

- The slope of the moment curve at any point is equal to the shear force at the same point.
- Points of zero shear are peaks of the moment curve, points of maximum positive or maximum negative moment.
- The change in moment value between any two points is equal to the area of the shear diagram between the same two points (provided no other external moment is applied). This is known as the Shear Area Method.

- The shapes of the curves from shear to moment always progress in this order; straight horizontal line, straight sloped line, parabolic curve and cubic curve. Therefore, if a shear curve is a horizontal line, the moment curve is a sloped line; if th e shear curve is a sloped line the moment curve is a parabolic curved line; etc. This happens because of the mathematical relationship between loading, shear and moment diagrams.

Example

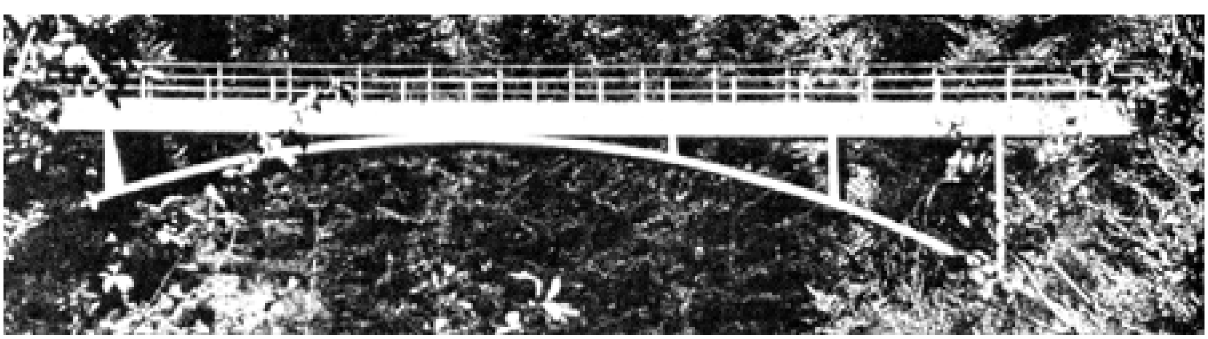


16V95

Lecture 37:

Development of Structural Form


The best works of structural engineering clearly exhibit a deep understanding of the way that forces move through a structure to reach their foundations. The bridges of Robert Maillart and Professor Dr. Christian Menn exemplify this knowledge. Maillart became known around the world for the three hinged arch of the Felsenegg Bridge, pictured above. He experimented with reinforced concrete in the manner of the great pioneers of the field. Structural designers around the world were investigating the ways in which reinforced concrete could be used in order to take advantage of its unique physical properties. For more information see *Reinforced Concrete* by Aly Ahmed Raafat (published by Reinhold in 1958).


This is the curved Schwandbach bridge. It is a deck-stiffened arch because the "U" profile of the road

deck is stiff enough that it can resist any deformations of the very thin supporting arch.

More to come.....

For more information on the Bridges of Robert Maillart, see the books of Prof. David Billington.

29IV95

Lecture 38:

Beam Stresses

BENDING STRESSES
A formula based on bending in a beam will be derived that can be used to:

- Select a beam that has adequate strength to carry a given load in bending.
- Determine the bending stresses in a given beam caused by a specific loading condition.
- Determine the greatest moment a given beam will resist (or the greatest load it will carry)

It will not be necessary to be able to derive the formula. However, it is essential to work through the derivation at least once in order to understand it and have confidence in how it is applied.

$$\mathbf{f = M/S \text{ or } S = M/F}$$

In which,

S is the Section Modulus in in^3. This beam property can either be calculated or read from tables for most beams.

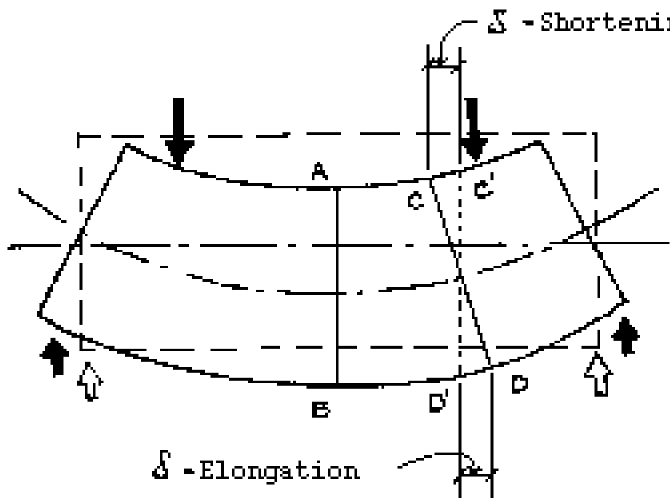
M is the moment in the beam in inch-pounds (or inch-kips). In the selection of a beam it is taken from the moment diagram.

F is the bending stress (psi or ksi) (F_b is usually used to denote the allowable bending stress; f_b is used to indicate the actual bending stress).

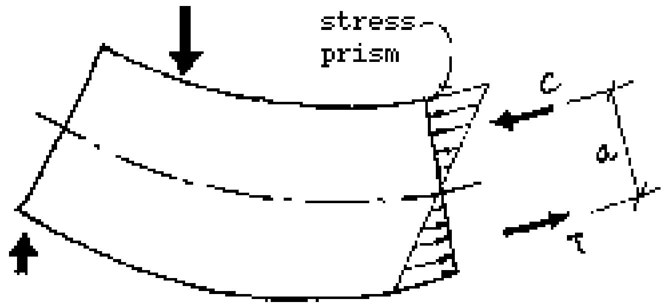
BENDING THEORY
There are a number of assumptions that were made in order to develop the *Elastic Theory of Bending*. These are:

- The beam has a constant, prismatic cross-section and is constructed of a flexible, homogenous material that has the same Modulus of Elasticity in both tension and compression (shortens or elongates equally for same stress).

- The material is *lineraly elastic*; the relaitonship between the stress and strain ar directly proportional.
- The beam material is not stressed past its proportional limit.
- A plane section within the beam before bending remains a plane after bending (see AB & CD in the image below).
- The neutral plane of a beam is a plane whose length is unchanged by the beam's deformation. This plane passes throught the centroid of the cross-section.
In order to visualize this, think of a black rubber beam with three lines drawn on its side. The dashed lines in the diagram below represent this beam, and the neutral axis and lines AB and CD are drawn parallel to their respective sides. Lines AB and C D are separated by some distance that is determined, but is not of consequence for the following discussion. These two lines were parallel before bending. As the beam bends, these lines remain perpendicular to the neutral axis.



Thus, as the beam bends, developing a curve in the neutral plane that reflects the bending, the line CD does not remain parallel to line AD. There is a distinct shortening at the top face of the beam and elongation at the bottom face. CC' is equal to d (delta), or the shortening of the top fiber. Similarly, DD' is equal to (delta) or the elongation of the bottom fiber (tension). If these were measured, they would be found to be equal, but opposite. Knowing that the Modulus of Elasticity (E) describes a linear relationship between the strain and the amount of stress in a material and that this material is homogeneous and has a certain value for E, we can determine the stress. The magnitude of the strain at any point along C'D' can be found by using similar triangles. Therefore, the stress is also proportional to the distance from the neutral plane, as illustrated in the following diargram.



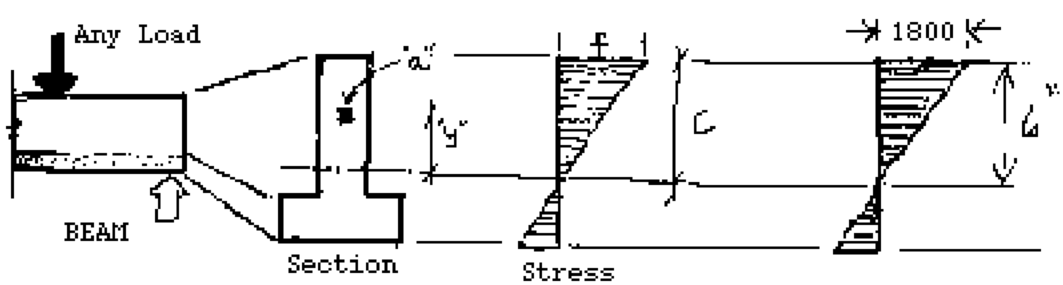
The loads and reactions acting on this beam segment cause a tendency for clockwise rotation (a clockwise moment). An internal moment counteracts this tendency so that the beam segment remains in equilibrium. Thus, the magnitude of the internal moment is exactly equal to the moment due to the external loads and reactions; but in the opposite direction. There is a state of equilibrium:

Internal Moment = External Moment

The illustration above shows the stress prisms developed due to the straining of the material. These prisms (compression and tension) can each be resolved into a single force that acts through the centroid of each prism. These two resulting forces (one is a compressive force (C) and the other a tensile force (T)) are found to have equal but opposite magnitudes. They are separated by a distance that is approximatly 2/3 of the depth of the beam (because of their triangular geometry) and create an *internal couple(!)*which causes the **internal moment resistance**of the beam. The exact location of the resultant forces of the stress prisms depends upon the locaton of the centroid of the entire stress prism. The location of the centroid is in turn dependent upon the exact distribution of the stresses across the section which this is determined by the geometry of the cross-section.

GENERAL CASE

If one accepts the assumptions given in the discussion of the bending theory above, the formula for determining the bending stress at any given fiber in a section can be derived as follows.



Given the beam as shown above loaded so that it has an internal bending moment. The stress at the extreme face (the distance C in this case) from the Neutral Axis (NA) is equal to Sigma or $\mathbf{f_b}$. The stress at any distance "y"; from the NA can be found using similar triangles. Thus,

$$f_y = (f_b/c)(y)$$

If "a" is a very small area at a distance "y" from the NA, then magnitude of the stress on area "a" equals $(f_b/c)(y)$. Remembering that stress is equal to a force distributed over an area, the force on the area "a" is equal to the stress times that area, or $(f_b/c)(y)$**(a)**. And, the moment of the force on area "a" about the NA is equal to $(f_b/c)(y)(a)$**(y)**.

Now, if all of the moments of all of the forces on all of the tiny "a" areas across the face of the beam are added together we arrive at an expression of the internal moment resistance of a beam;

M =Sum (f/c)ay^2 or **M = (f/c) Sum ay^2**

In this case, the value of (f/c) is a constant for any beam and Sum ay^2 is the second moment of the area, better known as the Moment of Inertia. Therefore, substituting items that we know, the equation becomes:

$$M = (f_b/c)\ I$$

or rearranging

$$M = (I/c)\ f_b$$

or

$$M = S^*f_b$$

(I/c) is known as the Section Modulus (S) and is also tabulated.

BENDING STRESSES REVIEW

The **flexure formula**:

M = (f/c) I or **M = (I/c) f**

> **M** is the internal moment in a beam created by the loads and reactions. It is numerically equal to the external moment at any section. Its value can be taken directly from the moments diagram. **M** is in inch-pounds (or inch-kips) .
>
> **f** is the stress at any distance "c" from the NA. If "c" is taken as the distance from the NA to the most remote fiber, then the stress "f" is the greatest stress. c is in inches; **f** is psi (or ksi).
>
> **I** is the Moment of inertia of the beam. I is in inches^4.
>
> **S** is the Section Modulus. It is **(I/c**.) and has the units of inches^3.

The flexure formula is useful in the following ways:

- $M = (F_b/c)\ I$ to find the resisting moment of a beam
- $f_b = M\ c\ /I$ to find the stress in a beam
- $I\ /\ c = F_b$ to select a beam
- $S_{req} = M\ /\ F_b$ to select a beam

The last formula is the most useful. The value of I/c, or S, is constant for any given beam and is utilized so often that its value is listed in any table of beam properties.

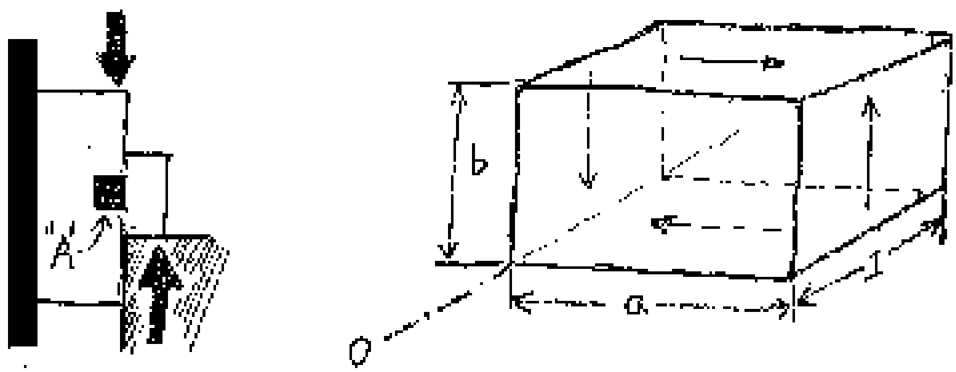
EXAMPLE PROBLEM Bending Stress in a Beam

EXAMPLE PROBLEM Bending Stresses

Lecture 39:

Shearing Stresses

For small members in shear, it is customary to assume shear is uniformly distributed over the entire area. However, this assumption is not permissible for beam cross sections. The vertical shearing stress at any point in a beam may be determined from the horizontal shearing stress at that cross-section.



- The infinitesimally small particle "A", whose dimensions are a, b, and 1, lies in a plane subject to shear.

- f_s represents the vertical shearing stress and f_v represents the horizontal shearing stress.

- $f_s b1$ is the total shearing force on one vertical surface (force is equal to stress f_s times the area b1); since the particle is in equilibrium an equal and opposite force must be acting on the other parallel face.

- Similarly, $f_v a1$ represents the total shearing force on each of the horizontal planes.

- The particle is in equilibrium, therefore the sum of the moments about any axis must equal zero. Take moments about axis "O":

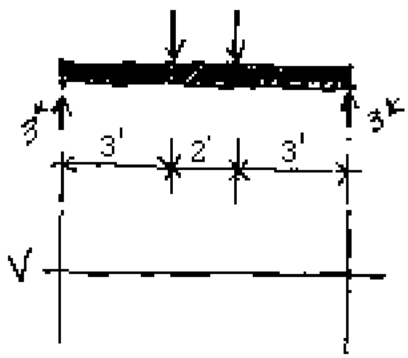
This shows that at any point, the horizontal shearing stress is equal to the vertical shearing stress.

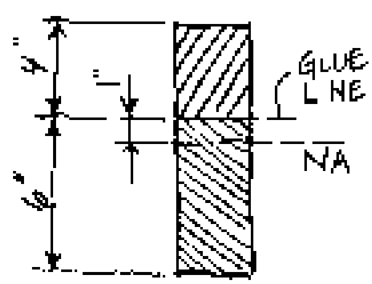
The general shear formula will find the shearing stress at any point in a beam. It is:

$$\mathbf{f_v = V\, A'\, y / I\, b}$$

V = the absolute value of vertical shear from the shear diagram at the point where shear is being investigated.
A' = the area above or below the horizontal plane being investigated.
y = the distance from the NA to the centroid of A'.
I = the moment of inertia of the **entire** section
b = the width of beam at the point where shear is being investigated.

Before deriving the formula, we will look at an application. If we wanted to find the maximum stress in the glue-line for this 2 x 10 composite beam:





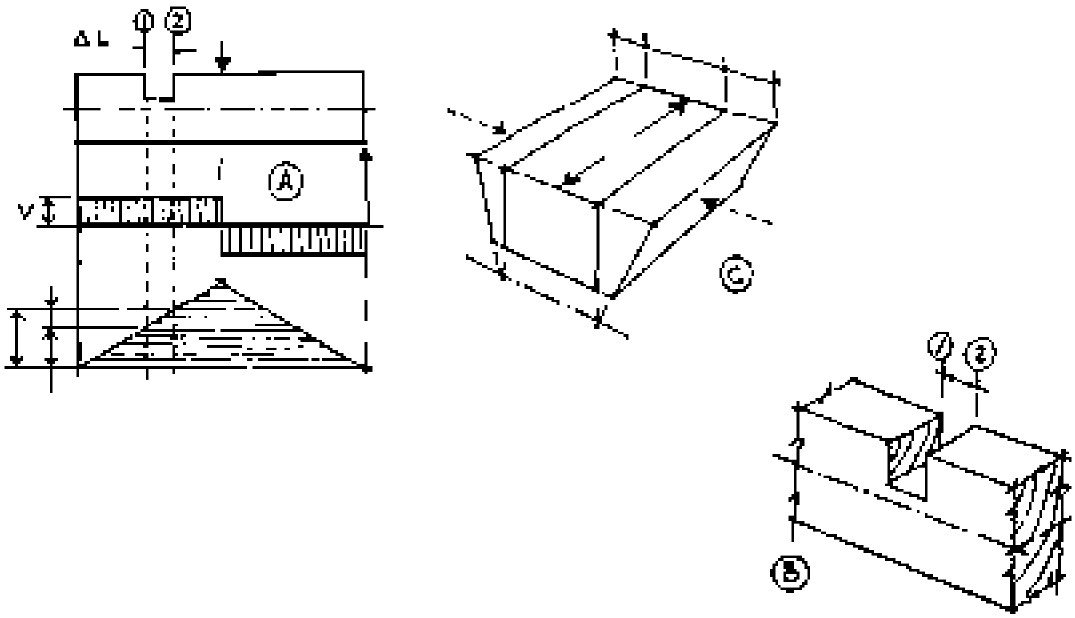
beam cross section

- First draw the V diagram.
- Next, determine the geometry and assiciated properties of the beam and the appropriate value for y.
- Then plug into the equation to determine the stress at the glue-line.
- In this case, the stress would be 3000# (4")(2")(3") / (1/12)(2")(10^3)(2") or 216 psi.

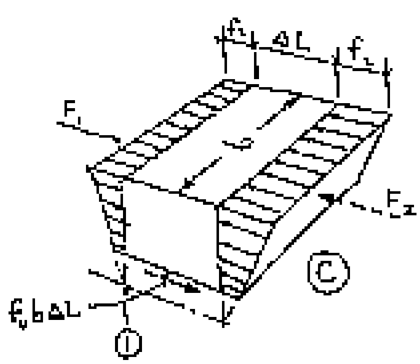
Note: This is not a *bending* stress, it is a **shearing** stress.

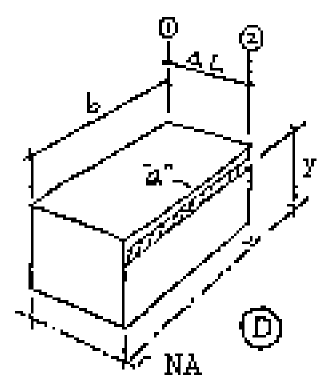
SHEAR FORMULA

Development of the general shear formula:



1. Figure "A" shows a loaded beam and its V & M diagrams.
2. Figure "B" represents a section of the beam with an infinitesimally small block of length Delta L removed.
3. Figure "C" shows the small block that has been removed from the beam. The bending moment is greater at section 2 than at section 1; so the stresses are greater at section 2 than at section 1. Therefore due to the difference in stresses the force F2 is greater than force F_1.
4. The small block is in equilibrium in the beam, therefore it must be in equilibrium as a free body; the sum of horizontal forces must be equal to zero. The horizontal shearing stress f_v acting over the area bDL is the third force, which when added to F_1 balances F_2. Therefore, $F_1 + f_v bDL = F_2$. (as shown in figure "C"), or $f_v bDL = F_2 - F_1$.





To develop the general shear formula express the forces F_1 and F_2 in terms of the bending moments:

5. From the flexure formula: f =
   or f_y =
6. Stress on area "a" at "y" distance from NA.: f_y =
7. Force on area "a" (F_2) equals stress times area: F_2=
8. Total force F_2 =
9. The first static moment of an area (from centroids):
10. Therefore, the **total** force F_2 =
    and F_1 =
11. Substituting in the formula developed in step 4: f_vbDeltaL = F_2 - F_1
12. From figure "A" we note that the change in moment (M) between two sections (1 & 2) is equal to the area of the shear diagram (V L) between the same two sections.
13. Combining steps 12 & 13 gives the general shear formula:

f_vbDeltaL =

f_v=

f_v=

Review of the most important aspects of the general shear formula:

$$\mathbf{f_v = V\ A'\ y\ /\ I\ b}$$

V is always the absolute value of the total shear (from the V diagram) on the beam at the section being investigated (this is usually the point where the shear is greatest).

A' is always the area on the face of the structural section on either side of the plane at which shear is being investigated. For most beam applications this is the area above (or below) a horizontal plane through the neutral axis of a beam.

y is always the distance from the neutral axis to the centroid of the area A'.

I is always the moment of inertia of the entire cross sectional area of the beam with respect to the neutral axis.

b is always the total width of the beam at the plane where the shear is being investigated (this is usually the width at the neutral axis).

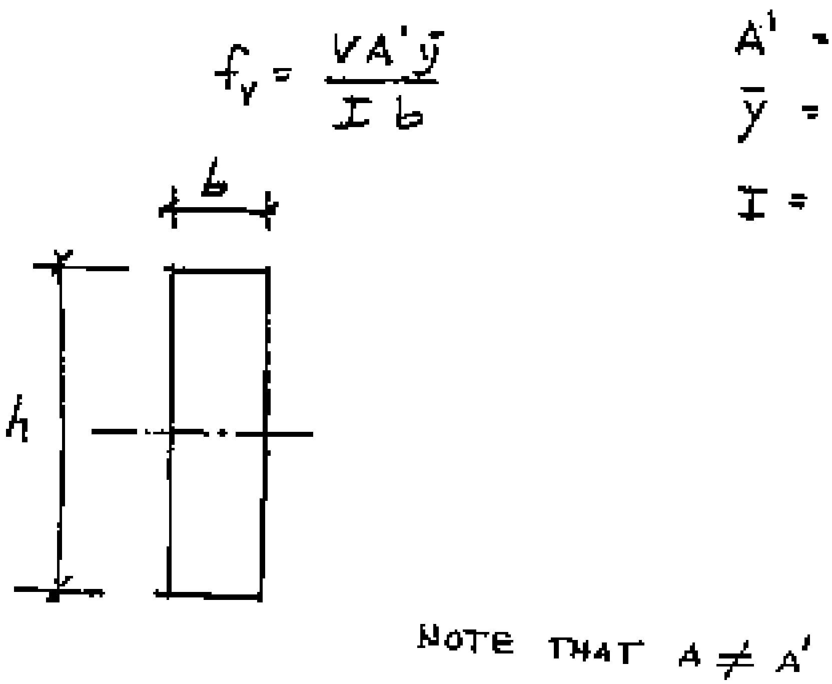
The general shear formula is often written with the static moment A'y represented as Q in which the general shear formula is:

$$f_v = V\,Q / I\,b$$

EXAMPLE PROBLEM Plotting Shear Forces

EXAMPLE PROBLEM Plotting Shearing Stresses

The second example illustrates that the maximum horizontal shearing stress in a rectangular beam occurs at the neutral axis. We can easily develop a formula for *this very special case* which is most useful for wood beams.



This formula applies **only** to the horizontal shearing stress in a **rectangular** beam and then **only** at the neutral axis.

IT APPLIES TO NOTHING ELSE!

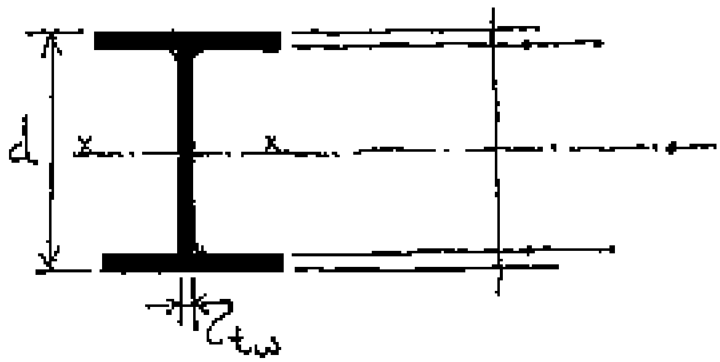
HORIZONTAL SHEAR FORMULA FOR A STEEL SECTION

The approximate web-shear formula recognizes that most of the shear stress in a steel section is taken by the web. It gives values within 10-15% of the actual true value from the general shear formula. (Fv = 14.5 ksi, A-36 steel)

$$f_v = V / t_w\,d$$

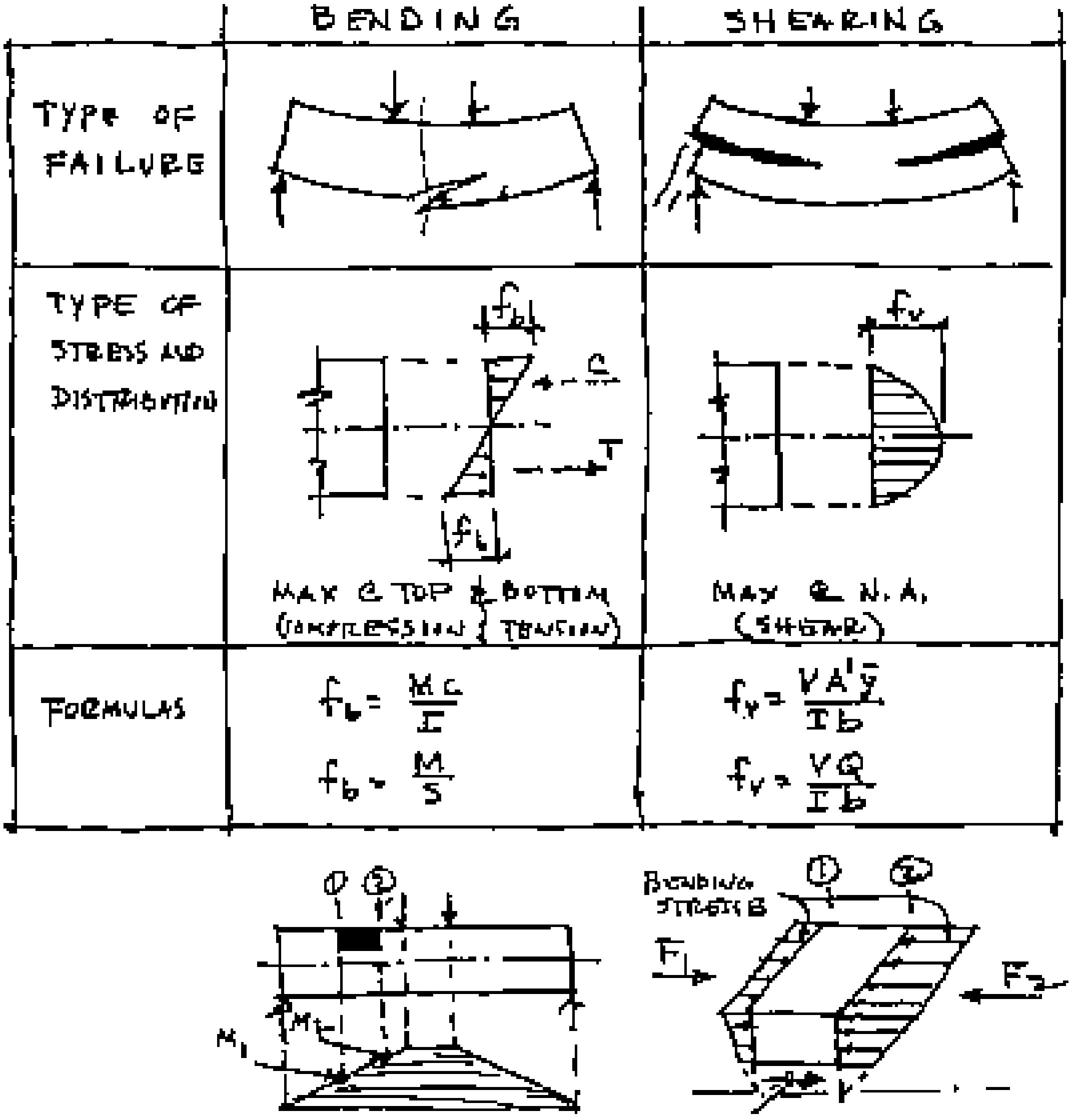
where

tw is the thickness of the web (read from the section tables)
d is the depth of the beam.



STRESS COMPARISON

The following is a comparison of bending and shearing stresses in a rectangular beam. It is an important table to memorize!


BENDING
SHEARING
TYPE OF FAILURE
TYPE OF STRESS AND DISTRIBUTION
C
T
MAX @ TOP & BOTTOM
(COMPRESSION & TENSION)
MAX @ N.A.
(SHEAR)
FORMULAS
$f_b = \frac{Mc}{I}$
$f_b = \frac{M}{S}$
$f_v = \frac{VA'\bar{y}}{Ib}$
$f_v = \frac{VQ}{Ib}$
BENDING STRESS
F_1
F_2
M_1
M_2

Lecture 40:

Beam Deflection

A loaded beam deflects by an amount that depends on several factors including:

- the magnitude and type of loading
- the span of the beam
- the material properties of the beam (Modulus of Elasticity)
- the properties of the shape of the beam (Moment of Inertia)
- the beam type (simple, cantilever, overhanging, continuous)

Deflection may or may not be critical. Excessive deflection may result in cracked plaster, objectionable appearance, or sags in flat roofs which then "pond". It may transfer loads to non- bearing members under the beam such as partitions, doors, windows, etc., which in turn may cause partitions to crack or doors and windows to stick. A floor beam which deflects excessively is apt to be "springy", creating an undesirable walking surface, even if it is in no danger of failing. A springy floor is especially unsatisfactory for a room housing sensitive instruments.

A general rule for limiting deflection of simple spans for floor construction, or for plastered ceilings, is that the deflection should not exceed the span (in inches) divided by 360 (max D= L/360). The deflection for exposed ceiling beams at the roof is often allowed to be 50% to 100% greater (l/240 or l/180). Codes usually specify that these deflections are based on live load only, but experience shows that this is sometimes excessive. A conservative approach is to limit the deflection to these values for total load in lightweight construction (such as wood and steel). These guidelines are general and apply in most cases, but certainly not all. For example, the springiness of a floor is influenced more by the mass of the floor than by the total deflection; more dead load could cause more deflection, but probably less springiness.

Formulas given in tables will be used to compute deflection for some loading conditions; these will be expanded to approximate deflections for other conditions. The deflection formulas will not be derived. In deflection formulas, **"w"** refers to pounds per inch of length of loading (**not** pounds per foot). **"W"** refers to the **total** distributed load. The beam length is in **inches**. Most mistakes in computing deflections are caused by using the length in feet instead of inches and/or using "w" to mean pounds per

foot instead of pounds per inch. Also, convert any distributed load "w" to "W".
There are four parts to deflection formulas:

1. **COEFFICIENT**, which takes into account:

- Type of beam
  simple, cantilever, overhanging, etc.
- Loading condition

2. **LOAD FACTOR**

- Distributed loads "W" (total weight)
- Distributed loads "w" (weight per **inch** of length)
- Concentrated loads "P" (weight of one concentrated load)

3. **LENGTH FACTOR**

- L^3 (inches) usually for load factor "P" and "W"
- L^4 (inches) usually for load factor "w"
- L_a, L_b (inches) used to locate some types of loads

(There is some confusion about the symbol **"L"** and **"l"**;. Lower case "l" is often used to indicate length in inches and upper case "L" for length in feet. However, lower case "l" can be confused with the number "1" which causes more serious problems. Therefore, in this class, "L" refers to inches in these deflection formulas.)

4. **STIFFNESS FACTOR**

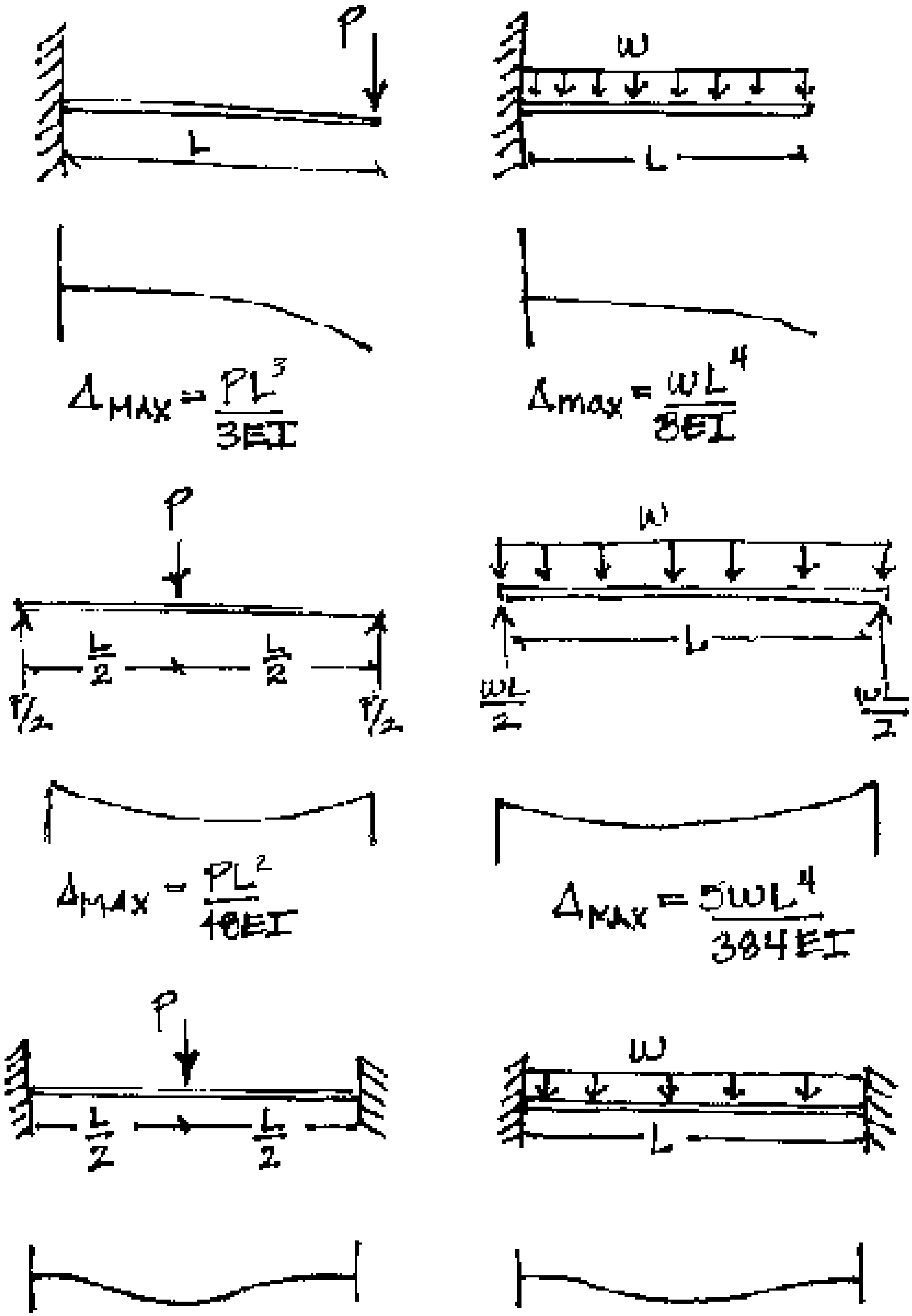
1/EI

- **E** = material stiffness (modulus of elasticity)
- **I** = stiffness of the section based on the geometry of the section (moment of inertia).

CAUTION: The most common mistake in computing deflection is caused by using "w" as load per **foot** instead of load per **inch**. The derivation of the deflection formulas uses the unit of inches for all the factors in the formula. Uniformly distributed beam loads "w" are normally described as load per foot

which must be converted to the proper load per inch value before insertion into the deflection formula. If "W" is used for the total distributed load instead of the load per unit of length ("w") this conversion is not necessary.
Note that W = wL; will simplify many of the formulas.

The following are equations for finding the deflection of the more common beam types and their associated loading. Note how the co-efficient reflects the stiffness of the system and the loading.



$$\Delta_{MAX} = \frac{PL^3}{192EI} \qquad \Delta_{MAX} = \frac{wL^4}{384EI}$$

EXAMPLE PROBLEM Beam Deflection

EXAMPLE PROBLEM Deflection Approximation

Lecture 41:

Beam Sizing

The following examples demonstrate methods for sizing beams based on principles that have been discussed up to this point. Actual structural design involves many other concepts that will be covered in following courses.

EXAMPLE PROBLEM Example Problem: Beam Sizing #1

EXAMPLE PROBLEM Example Problem: Beam Sizing #2

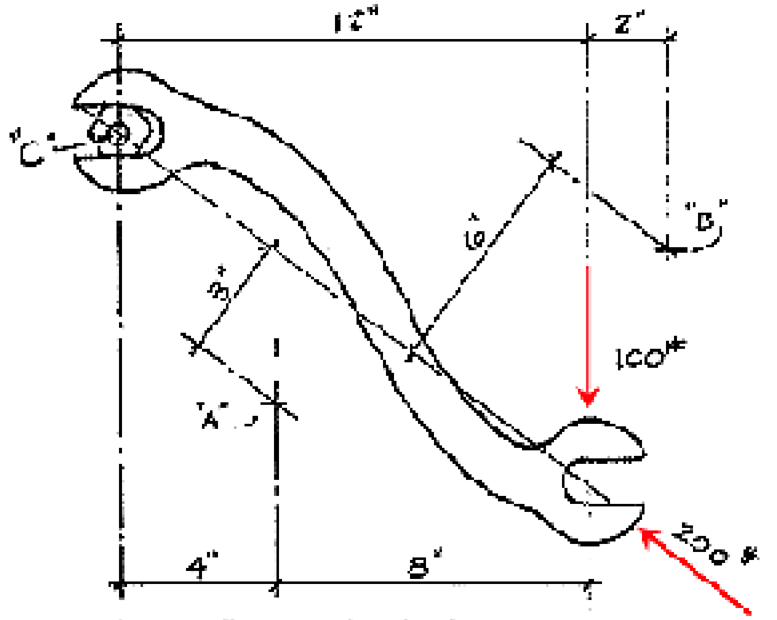
EXAMPLE PROBLEM Example Problem: Beam Sizing #3

EXAMPLE PROBLEM Example Problem: Section Efficiency

Lecture 5
Example Problem

Calculating a Combined Moment

Given:
Wrench shown below with two forces acting upon it
Determine:
the combined moment for both forces about point C



Solution:
In order to simplify the solution, begin by considering only one force at a time with **C** as the center of moments. In order to do this, consider that a moment is defined as the product of a force and a specific perpendicular distance. Find the moment for each force separately. The 100 pound force has a moment arm (perpendicular distance from line of action to the center of moments) of 12 inches. Therefore, the moment caused by this force equals

(100 pounds)(12 in) = 1200 pound-inches

The moment arm for the 200 pound force is zero because the line of action of the force passes through C. So the moment caused by the 200 pound force is

(200 pounds)(0 inches) = 0 pound-inches

In order to find the combined moment of the two forces, simply add their individual moments or

1200 pound-inches + 0 pound-inches = 1200 pound-inches

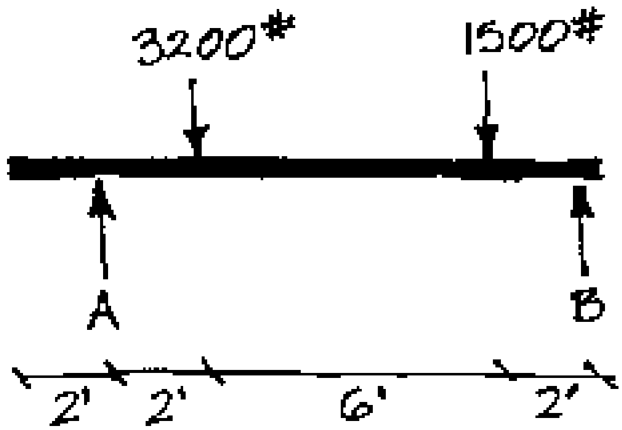
The moment resulting from these two forces can be taken about any point, not just the nut around which the wrench is physically turning. Take point A and repeat the above proceedure. The solution will be (100 pounds)(8inches) = 800 pound-inches and (200 pounds)(3inches) = 600 pound-inches. The total combined moment about point A which will be 800 pound-inches + 600 pound-inches = 1400 pound-inches

What is the total combined moment about point B?

Lecture 5
Example Problem

Moments on a Beam

Given: a beam with the geometry and loading indicated
Determine: the moment of the loads on this beam using the reaction at B as the center of moments



Solution: Moments can be used to determine the reactions at the ends of simple beams. Either point A or point B could be chosen as the center of moments. Since the exercise indicated that B is the center of moments, the first step is to determine all of the loads that will tend to cause a counter-clockwise rotation about B. These loads are placed on the left side of an equation and all of the loads which will tend to cause a clockwise rotation will be placed on the right side. The 1500 pound and the 3200 pound loads cause counter-clockwise rotations. The reaction at A is the only load that causes a clockwise rotation to the beam. Notice that the reaction at B does not even come into consideration. Why is this?

(1500 pounds)(2 feet) + (3200 pounds)(8 feet) = A x 10 feet

3000 pound-feet + 25600 pound-feet = A x 10 feet

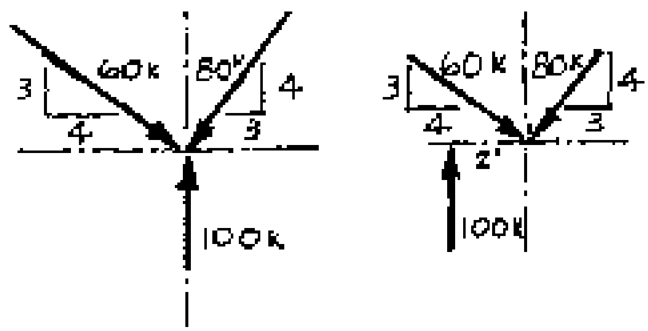
This yields a reaction of 2860 pounds at point A.

Try this if point A had been chosen as the center of moments. What is the reaction at B?

Lecture 6
Example Problem

Equations of Equilibrium

Given:
two force systems as
Determine:
whether or not equilibrium has been satisfied.



Solution:
We note that the system on the left is a concurrent system and the one on the right is a non-concurrent, non-parallel system. In order for either system to be in equilibrium, all three equations of equilibrium,
Sum $F_x = 0$,
Sum $F_y = 0$ and
Sum $M = 0$ must be satisfied.

Begin by determining if the sum of the forces in the vertical and horizontal directions are equal to zero. The simplest way to solve these two systems would be to break each of the diagonal forces into their vertical and horizontal component parts. One can utilize trigonometry functions for this or, from observation, each diagonal is the "5" side of a 3-4-5 triangle. Therefore, the side marked "3" has a value of 3/5 of the value of the diagonal and the side marked "4" is equal to 4/5 the value of the diagonal.

Now, using the components, solve for the sum of forces equations.

Sum F_x = 4/5 (60k) - 3/5 (80k) = 48 - 48 = **0**

Sum F_y = 100k - 3/5 (60) - 4/5 (80) = 100 - 36 - 64 = **0**

Both systems satisfy the sum of forces equations for equilibrium.

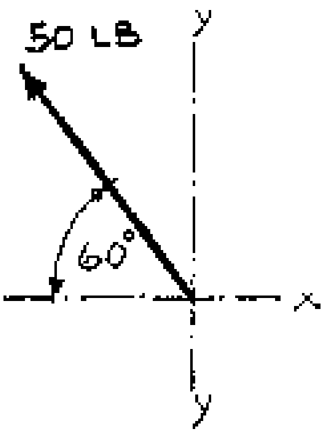
Now solve for the sum of moments equation. The system on the left is in moment equilibrium because it is a concurrent force system. Take the sum of the moments at their point of intersection. For each force, the moment arm is equal to zero. Once equilibrium has been established using this single point, the sum of the moments for that force system will be zero for any point on that plane.

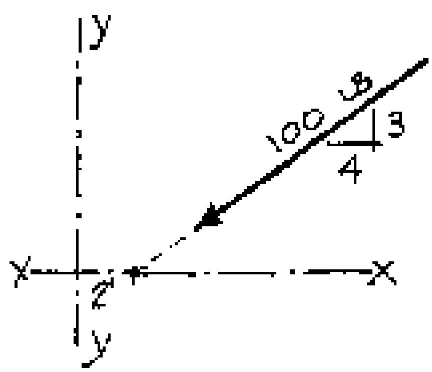
The force system on the right is **not** in moment equilibrium. Taking the sum of moments around the same point as before, the moment arm of the two diagonal forces are zero, but the 100 pound force will cause a clockwise rotation. This system cannot be put into equilibrium with a single force because that would disrupt the sum of forces equations. The easiest solution would be an applied moment, equal in magnitude to that caused by the 100 pound force, but opposite in sense.

Lecture 8
Example Problem

The Triangle Method

Given:
the force system shown below.
Determine:
the rectangular components F_x and F_y (H and V) by inspection and similar triangles; confirm this with a graphical solution. Note: Use a scale of 1" = 50 lb. (Drawings shown below are NOT to scale.)





Solution:
Both forces belong to easily recognizable triangles.

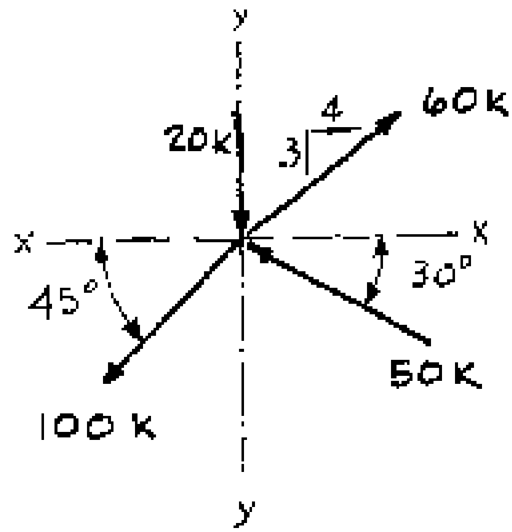
The system on the left is given away by its 60 degree angle. It is a 1-2-sqrt3 triangle. F_x = 1/2 F or 25# and F_y = (sqrt3)/2 F or 43.5#

The system on the right can be solved by inspection realizing that the triangle is the familiar 3-4-5. So, 100/5 = F_y/3 = F_x/4; thus, F_y = 60# and F_x = 80#.

Both systems can be verified through strictly graphic methods. Carefully draw the force to scale and then the horizontal and vertical components in the "head to tail" fashion of the triangle method. These components are then simply measured. The accuracy of the results rely upon the accuracy of the drawing.

Lecture 9
Example Problem

Algebraic Force Resolution



Given:
four concurrent forces with the magnitudes and geometry shown.
Determine:
a. the V and H components of each of the forces
b. the resultant algebraically
c. the equilibrant.

Solution:
a. First break each of the forces into its horizontal and vertical components, either by inspection or using the algebraic method. The components can neatly be entered one by one into a chart similar to the one shown below.

Force	Fx	Fy
60	-	-
50	-	-
100	-	-
20	-	-

The 20k force is the most straightforward, so enter its components into the chart first. Be careful to enter the correct sign for each component or the overall result will be incorrect.

Force	Fx	Fy
60	-	-
50	-	-
100	-	-
20	**0**	**-20**

The 60k force can also be resolved by observation because of its 3-4-5 geometry. $F_x = 4/5(60k) = 48k$ and $F_y = 3/5(60) = 36$. Enter these numbers into the chart.

Force	F_x	F_y
60	**48**	**36**
50	-	-
100	-	-
20	0	-20

The last two forces are most efficiently resolved using the algebraic equations:

$F_x = F \cos ø$
$F_y = F \sin ø$

So, for the 50k force, its components are

$F_x = -50k \cos 30 = -50(.866) = -43.3k$
$F_y = 50k \sin 30 = 50(.5) = 25k$

and for the 100k force

$F_x = -100k \cos 45 = -100(.707) = -70.7k$
$F_y = -100k \sin 45 = -100(.707) = -70.7k$

Use these components to complete the chart.

Force	F_x	F_y
60	48	36
50	**-43.3**	**25**
100	**-70.7**	**-70.7**
20	0	-20

b. The components of the resultant are equal to the sum of the columns of the completed chart

Force	F_x	F_y
60	48	36
50	-43.3	25
100	-70.7	-70.7
20	0	-20
R	**-66.7**	**-29.7**

The resultant is found using the Pythagorean Theorem:

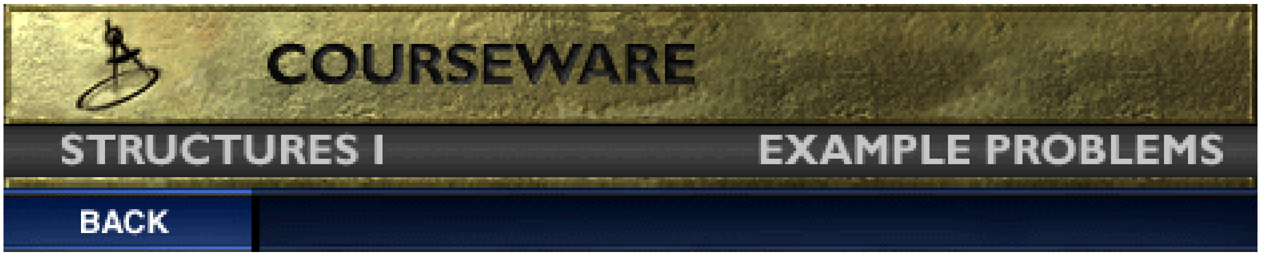
= SQRT (sum F_x^2 + sum F_y^2)
= SQRT (-66^2 + -29.7^2)
= SQRT (4356 + 882)
= SQRT (5238)
= **72.4k**

Its angle can be found by applying

tan ø = opposite side [F_y] / adjacent side [F_x]
tan ø = -29.7 / -66
tan ø = .45
ø = arctan .45
ø = **24.2 degrees below the x axis down and to the left**

The quadrant can be found by observation using the signs of the components of the resultant. In this case, both F_x and F_y are negative, so the resultant will lie in the lower left quadrant.

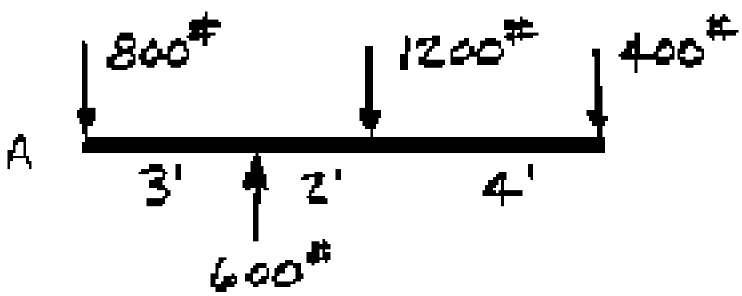
c. The equilibrant will be the inverse of the resultant. Its magnitude will be the same, 72.4k, but it will act at an angle 24.2 degrees above the x axis, up and to the right.



Lecture 10
Example Problem

Parallel Force Systems

Given:
the coplanar system of forces shown below.
Determine:
the resultant (this includes the Magnitude, Direction and Sense, and a Point on its Line of Action); AND, the force(s) required to put this system into equilibrium.



Solution:
Since there are only vertical forces in this system, only the sum of the vertical forces will have to be determined in order to find the magnitude of the resultant of the system.

Sum F_y = Resultant = -800# + 600# - 1200# -400# = -1800#

The resultant is -1800# or 1800# down. The location of the resultant must be determined through the moment equilibrium equation. In this case take the sum of the moments around A for simplicity, although any point would yield the same result.

```
Sum MA = 1800# x = -600#(3ft) + 1200#(5ft) + 400# (9ft)
                 = -1800#ft + 6000#ft + 3600#ft
                 = 7800#ft
                x = 7800#ft / 1800# = 4.33ft
```

This distance can be checked by taking the moment around another point. (Though not the point through which the resultant acts) Take, for example, the other end of the beam. The distance for the resultant should be

9ft (total length of the beam) - 4.33ft (distance from A)
= **4.67ft** (distance from the other end)

```
Sum M = -1800#(4.67ft) = -1200#(4ft) + 600#(6ft) - 800#(9ft)
              -8406#ft = -4800#ft + 3600#ft - 7200#ft
              -8406#ft = -8400#ft *
```

*Since the distance 'd' is a repeating decimal, the amount of discrepence will depend upon the number of decimal places used. In this case, these numbers are sufficient to confirm the location of the resultant.

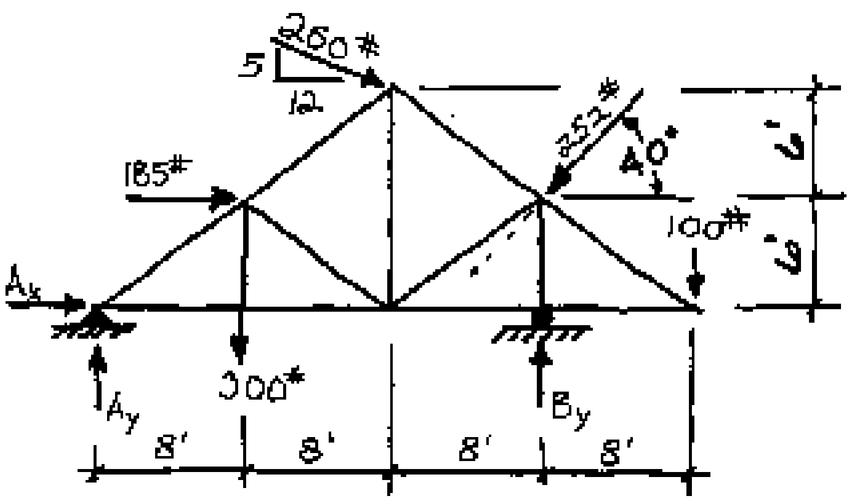
Knowing the magnitude, direction and location of the resultant, the equilibrant is found by simply inversing the direction of the resultant. The equilibrant acts with a force of 1800# up at a point 4.33 ft to the right of point A.

29IV95CHL

Lecture 10
Example Problem

Principle of Moments

Given:
the following truss with the loading indicated
Determine:
the moment of each of the loads about point A; the combined moment of all the loads about point A; the required reaction at B to satisfy equilibrium



Solution:
First find the moment caused by each of the loads around point A, which will equal the force, or its components, multiplied by the moment arm. Clockwise moment will be posative.

Moving from left to right through the truss:

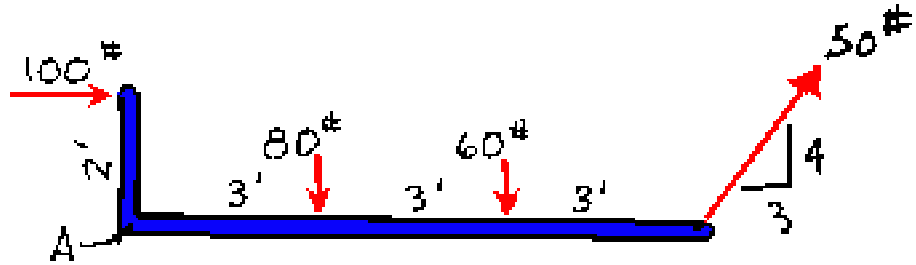
(185#)(8ft) = 1480#-ft
(300#)(6ft) = 1800#-ft
(260#)(12/13)(12ft) = 2880#-ft
(260#)(5/13)(16ft) = 1600#-ft
(-252#)(cos 40)(6ft) =

(252#)(sin40)(24ft) =
(100#)(32ft) = 3200#-ft

Lecture 11
Example Problem

Algebraic Resolution of a Non-Concurrent System

Given:
the system of forces
Determine:
the resultant by an algebraic method (first its magnitude, and then the point on its line of action where it meets the crooked arm)



Solution:
The first step is to determine a convenient location for the center of moments. This point should be chosen with care since it can make the problem much easier if correctly chosen. Assume that point A shall be taken as the center of moments. Next, inspect the forces in order to find any that must be decomposed into their components. Only the 50 pound force must be decomposed into its orthagonal components. It has a line of action that is a 3-4-5 triangle. This can be clearly seen by observation. Thus, the components are F_x = 30 pounds and F_y = 40pounds.

Next, find the magnitude and direction of the resultant by summing the moments of the forces and/or their components.

Sum F_x = 100 pounds + 30 pounds = 130 pounds
Sum F_y = -80 pounds - 60 pounds + 40 pounds = -100 pounds
F = SQRT ((130 pounds)^2 + (-100 pounds)^2)) = **164 pounds**

Next, the direction must be determined.

Tan ø = -(100/130), so ø = 37.6 degrees below the x axis and to the right

Then, sum of the moments around point A to find the location of the resultant. Assume that the resultant acts along the long side of the crooked arm so that Fx will cause a zero moment.

Sum M_A

(100 pounds)(x ft) = (100 pounds)(2ft) + (80pounds)(3ft) + (60 pounds)(6ft) - (40 pounds)(9ft)
100x pound-ft = 200 pound-ft + 240pound-ft + 360pound-ft - 360 pound-ft
100x pound-feet = 440 pound-feet
x = 4.4 feet

Therefore, the resultant equals 164 pounds acting 37.4 degrees below the x axis, down and to the right at a point 4.4 feet to the right of point A.

Lecture 14
Example Problem

Free Body Diagram and Reactions of a Beam

Given:
the beam and loading as shown.
Determine:
the magnitude of the reactions at A and B after drawing a FBD of the system.

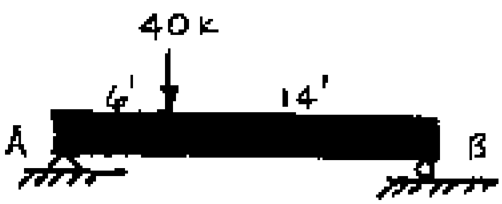
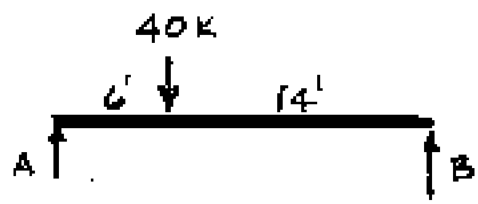


Diagram of a Loaded Beam



The FBD of the Same Beam

Solution:
The reactions at A and B are replaced by forces at A and B. The force at B must be vertical because its roller support can only react perpendicular to the surface upon which it rests. The line of the 40k load and the reaction at B have no horizontal components so it should be clear that the horizontal component of A must be zero. Therefore, it is not shown. If we assumed a horizontal component at A and showed it on the FBD, we would find through the "sum of the horizontal forces equals zero" that the horizontal component was zero. It would still be a valid FBD. Everything that is needed to solve the FBD is shown.

Now use the FBD and the principles of equilibrium, "sum of the forces must equal zero" and "sum of the moments must equal zero," to solve for the unknown forces.

If we try to solve for the vertical forces, we have two unknowns and only one equation. If we track moments about the line of action of either A or B, we will have a moment equation with only one unknown. For this reason, points A or B are convenient points around which to take the moments.

If we take the moments around point A, Sum $M_A = 0$, we know that the absolute value of the counter-clockwise moments must equal the absolute value of the clockwise moments.

Sum M_A = 0
40 kips)(6 feet) - (B)(20 feet)
240 kip-feet - (B)20 kip-feet
(B)(20 kip-feet) = 240 kip-feet
B = 12 kips

The reaction at point A may now be found either by taking the moments around point B, Sum M_B= 0, or by summing the vertical forces, Sum F_y= 0, and using the reaction at B found above.

Sum M_B = 0
(-40 kips)(14ft) + (A)(20ft)
-560 kip-feet + (A)(20 kip-feet)
(A)(20 kip-feet = 560 kip-feet
A = 28 kips

To check:

Sum F_y = 12k + 28k - 40k = 0

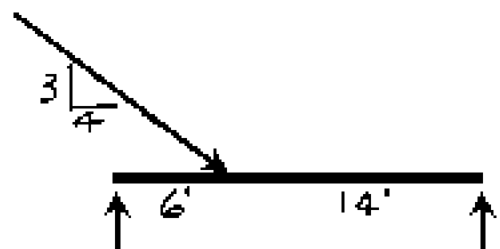
Lecture 14

Example Problem

Components of Reactions

Given:
the beam of the previous example problem with the loading modified to contain a horizontal component as seen in the figure below.
Determine:
the reactions at A and B.

Loaded Beam



Free Body Diagram

The horizontal component can be resisted only by the reaction at A since the support at B is a roller support. The pinned reaction at A will resist translation in any direction. The assumed reaction A is replaced by its rectangular components A_x and A_y (or A_H and A_V). The reaction at A is a single force; A_x and A_y represent the components of that one force.

If we do not know the direction of the force, we must assume a direction. If the assumption is correct, the answer will have a positive sign; if the assumption is incorrect, the answer will have a negative sign. Although in this case it is possible to g uess the sign of the reactions, it is not always so. Choosing "all posative"- up or to the right- minimizes possible confusion over the meaning of resulting signs.

Use the three equations of equilibrium to solve for the reactions.

Sum $F_x = 0 = 4/5(40k) + A_x$
Sum $F_y = 0 = -3/5(40k) + A_y + B$
Sum $M_A = 0 = 3/5(40k)(6ft) - B(20ft)$

The first and third equations can be solved immediately.

A_x = -32k
B = 7.2k

Now use B = 7.2k in the second equation to solve for A_y.

Sum F_y = 0 = -24k + 7.2k + A_y
A_y = 16.8k

A can be found using the Pythagorean Theorem.

A = SQRT (-32k^2 + 16.8k^2)
A = 36k

The direction is up and to the right. The angle above the x axis equals

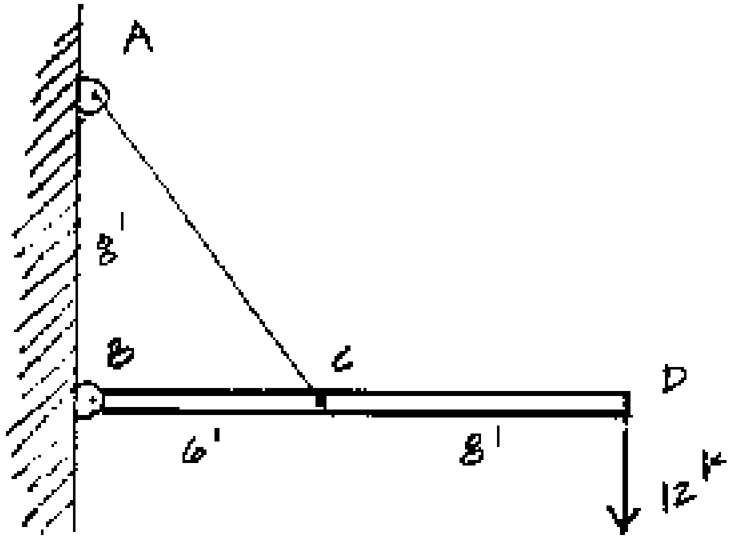
ø = arctan (-16.8 / 32) = 27.7 degrees

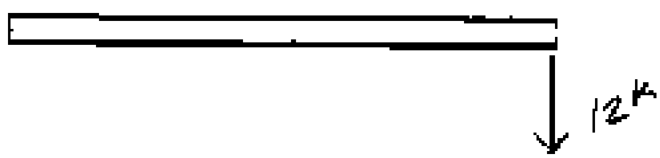
In summary, the reactions of the beam are as follows:
at A a force of 36 kips acting up and to the right at an angle of 27.7 degrees and
at B a force of 7.2k acting up.

Lecture 15
Example Problem

Three-Force Members

Given:
the system with the loading indicated below.
Determine:
a. the FBD of the horizontal element using the images below
b. the reactions at B and C algebraically
c. the reactions at B anc C graphically





Solution:

a. The free body diagram of the horizontal member will include the 12k force at D and the reactions at B and C. The support at B will have a horizontal and a vertical component although the angle at which the total force B acts is unknown at this point. The direction of C is already known because it must act in the direction of the wire, but it is more easily solved when it is broken into its rectangular components.

b. Apply the familiar equations of equilibrium to algebraicly solve for the reactions at B and C.

Sum $F_x = 0 = B_x + C_x$

$\text{Sum } F_y = 0 = -12k + B_y + C_y$
$\text{Sum } M_B = 0 = -C_y(6ft) + 12k(14ft)$

Solve for C_y using the moment equation:

$6C_y = 168kft$
$\mathbf{C_y = 28k}$

Solve for B_y using C_y in Sum $F_y = 0$:

$-B_y = -12k + 28k$
$-B_y = 16k$
$\mathbf{B_y = -16k}$

Now there are two more unknowns, B_x and C_x, but only one other equation. However, from observation, C is a 3-4-5 triangle. Using this information, solve for C and C_x:

$C_y = 28k = 4/5\ C$
$\mathbf{C = 22.4k}$
$C_x = -3/5\ (22.4k)$
$\mathbf{C_x = -13.44k}$

Now solve for B_x:

$B_x = -C_x$
$\mathbf{B_x = 13.44k}$

Finally, solve for B using the Pythagorean Theorem:

B = SQRT (13.44k^2 + (-16k)^2) = SQRT (436) = **20.9k** acting down and to the right

The angle for the reaction at B would be:

ø = arctan (-16 / 13.44)
ø = 53.5 degrees below the x axis

c. To solve for the reactions graphically, remember that any one force in a three-force system must be the resultant of the other two forces. This must be so for the element to remain in equilibrium. Additionally, if the system is in equilibrium and if the forces are not parallel, they must be concurrent.

Now, extend the lines of action of the two forces that have a known direction until they intersect. This is the point of origin for the three-force system. It is a VERY important point! Now, the ine of action and the direction of the reaction at B can be found by simply drawing a line from point B to the point of origin. This is the line of action of the force. The direction of the resulting third force is also known when inspection determines what kind of force must be applied to put the system into equilibrium. The magnitude of B and C can be found using the triangle method and making assumptions about the sense of the forces.

COURSEWARE

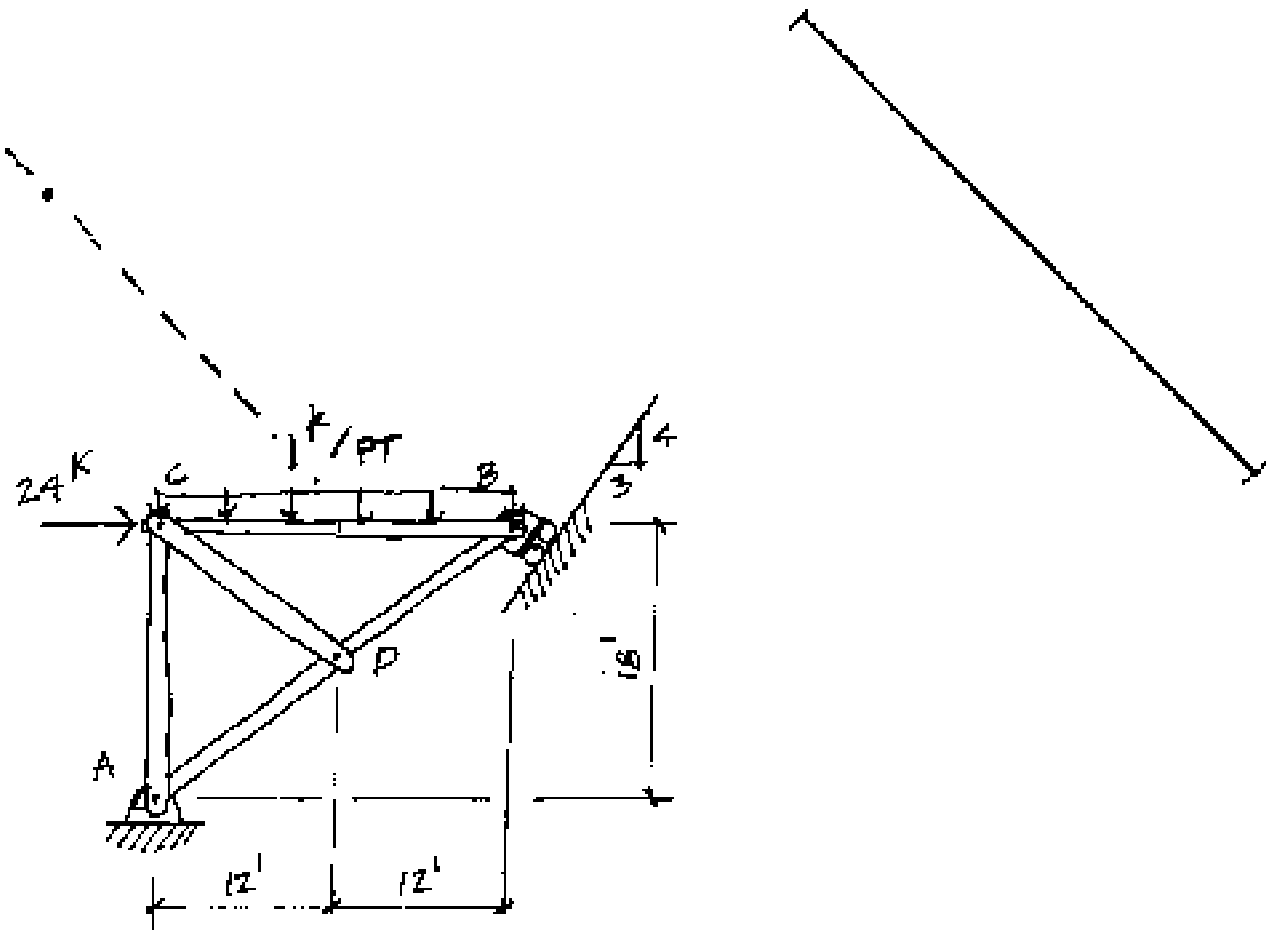
STRUCTURES I EXAMPLE PROBLEMS

BACK

Lecture 16
Example Problem

System Reactions

Given:
the system and loading as shown.
Determine:
a. the FBD
b. the reactions at A and B analytically



Solution:

a. The system FBD will include the known concentrated and distributed loads, 2 reactions at A (A_x and A_y) and a single reaction at B, perpendicular to the surface supporting the roller. Note that this surface has a 3-4-5 geometry.

b. Write the equations of equilibrium breaking the reaction at B into its rectangular components.

Sum $F_x = 0$

$A_x - B_x + 24$

Sum $F_y = 0$

$A_y + B_y - 24$

Sum $M_A = 0$

24k(18ft) + 1k/ft(24ft)(12ft) - B_y(24ft) - B_x(18ft)

Watch the signs! Realize, in this case, if B_y is assumed to be positive, B_x **must** be negative and vice versa because both components must either act toward the diagonal surface or away from the diagnonal surface.

At this point, all three equations have two variables making them impossible to solve. However, remember the 3-4-5 geometry of the reaction at B. Therefore, B_x = 3/5 B and B_y = 4/5 B. By replacing these values in the moment equation, it is possible to solve for B.

Sum M_A = 0
24k(18ft) + 1k/ft(24ft)(12ft) - 4/5B (24ft) - 3/5B (18ft)
432kft + 288kft -19.2B - 10.8B
720kft - 30B
30B = 720 kip-feett
B = 24 kips

Now use the force equations to solve for Ax and Ay.

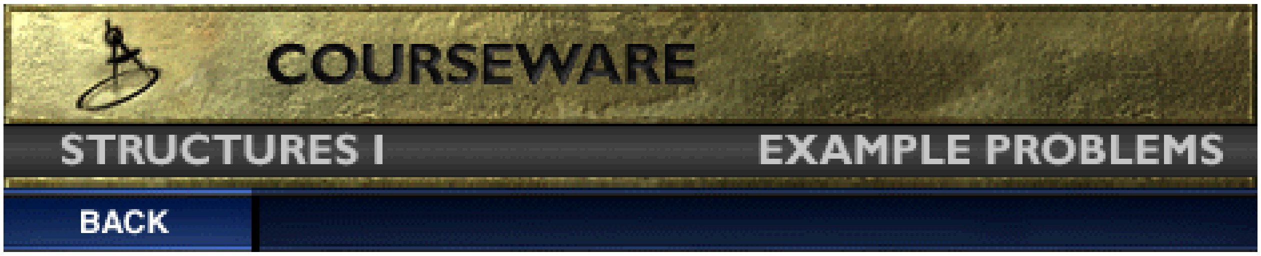
A_x - 3/5B + 24 = 0
A_x - 14.4 + 24 = 0
A_x = -9.6 kips

A_y + 19.2 - 24 = 0
A_y = 4.8 kips

Finally, find the total reaction at A and its directional angle.

> A = SQRT (-9.6^2 + 4.8^2) = 10.73k
> ø = arctan (-4.8/9.6) = 26.6 degrees up and to the right.

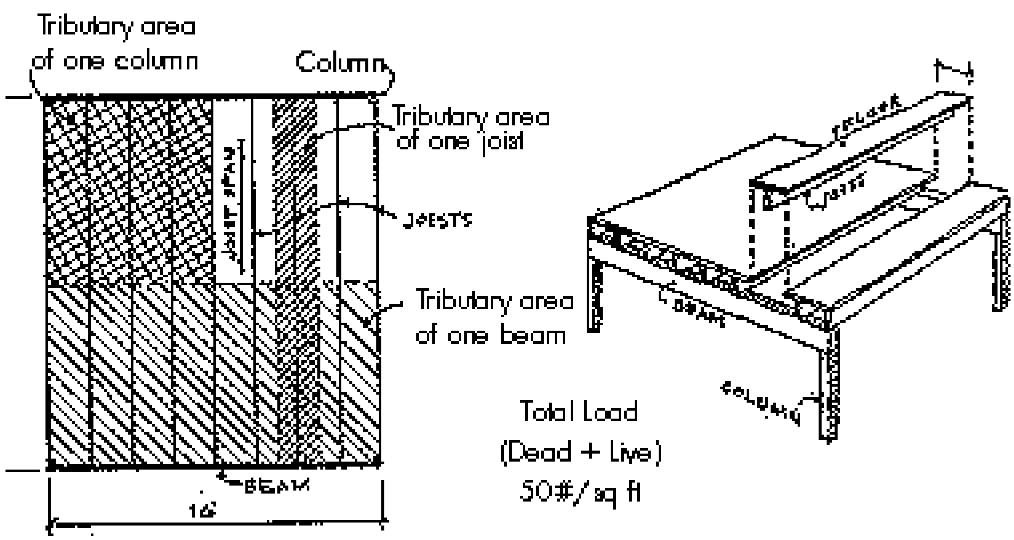
In summary, the reaction at A equals 10.73 kips acting at an angle of 26.6 degrees up and to the right and the reaction at B equals 24 kips acting perpendicular to the diagonal surface.



Lecture 19
Example Problem

Load Distribution

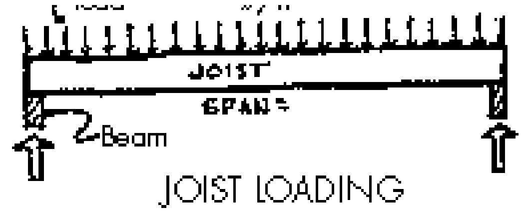
Given:
the floor framing diagram below with a total design load of 50 psf (10 psf Dead Load + 40 psf Live Load); the floor joist spacing is 2 feet on center; the joists are 18 feet long; the beams are 16 feet long
Determine:
the total load on each of the columns.



Solution:
The tributary area of one joist is the area supported by that joist; the area equals half the distance to the adjacent joists to either side multiplied by the full length of the joist. The joists rest on beams and provide the load on the beams. In this example, each beam supports one end of each joist (usually beams support joists on each side, rather than on only one side as shown in this example.) The area contributing to the beam load is half the area supported by the joists. The beams are supported in turn by columns; for this problem, the tributary area of a column is half the area supported by a beam.

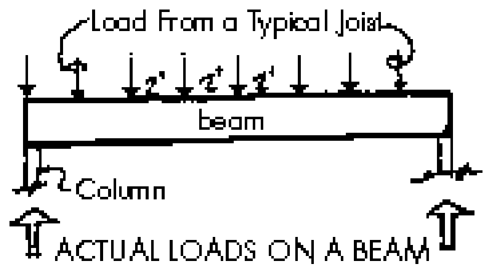
Each foot along the length of a joist supports an area spanning

two feet. This is the center to center spacing of the joists.

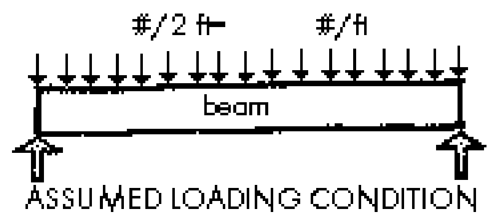


The load on a one foot length of a joist is 50 psf x 2 sqf/ft = 100 plf (pounds per linear foot). This is the design load on the floor multiplied by the tributary area for the joist per linear foot. Therefore, the total load supported by one joist equals the linear load multiplied by the length of the joist or 100 plf x 18 feet = 1800 pounds.

Since the joists are supported by the beams, the sum of the reactions at the ends of the joists is the total load on the beams. Since the joists are symmetrically loaded, they distribute their load equally to the supporting beams at either end. The concentrated load on a beam from one joist is 900 pounds every two feet. Loads from the joists on their supporting beams are usually considered to be uniformly distributed if there are five or more equally loaded and spaced joists.



Assume that the load in this example is uniformly distributed. What is the load/foot on the beam? The load on the beam is equivilent to the total of each point load contributed by the joists "spread" out over the length of the beam. That is, 900 pounds every 2 feet for 16 feet which is a total load of 900 pounds x 8 = 7200 pounds. If this was distributed over the entire beam, it would be a linear load of 450 pounds/ft.



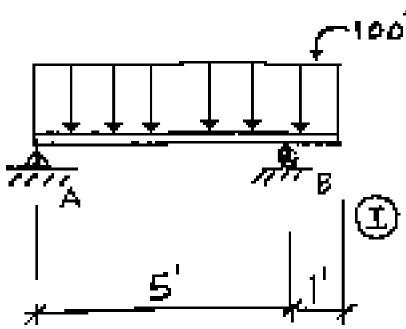
What is the load on a column?
The load on the column has the same value as the reaction at the beam end. The reaction from the element above becomes the action on the element below. Thus, the load on the column is 3600 pounds; 7200 pounds divided by two.

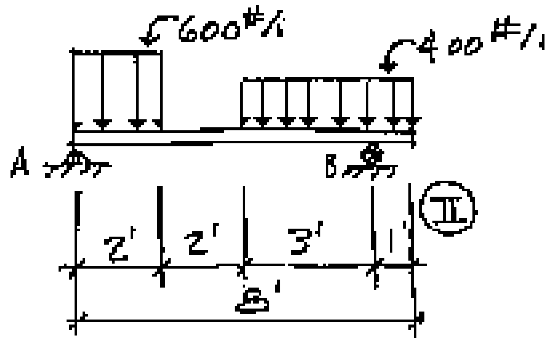
COURSEWARE
STRUCTURES I EXAMPLE PROBLEMS
BACK

Lecture 20
Example Problem

Distributed Loads

Given:
the uniformly distributed loads indicated below.
Determine:
the resultant load for each of the uniformly distributed loads.





Solution:
Both systems are uniformly distributed loads. To find the resultant of a uniformly distributed load simply multiply the magnitude of the load in #/ft by the length of the load. The line of action of the resultant of a uniformly distributed load will alw ays act through the midpoint of the original load.

I. The resultant of the loads of the system at the left are found by taking the load of 100 #/ft and multiplying it by the total length of the beam, 6 feet. This yields a total of 600 pounds of total load and acting down through the midpoint of the lengt h or:

100#/ft (6ft) = 600# acting 3 ft from A

II. The problem on the right can be solved by considering each distributed load separately. The resultants for each of the loads is found first, and then they can combined to determine the resultant of the system. First,

600#/ft (2ft) = 1200 #

This force acts 1 ft to the right of A.
Second,

400#/ft (4ft) = 1600#

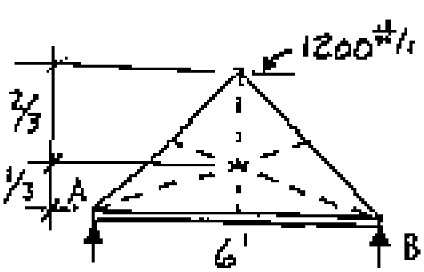
This force acts at 1ft to the left of B or 6 ft from A.
These forces can be combined in ways discussed in earlier examples to find the resultant for the system.

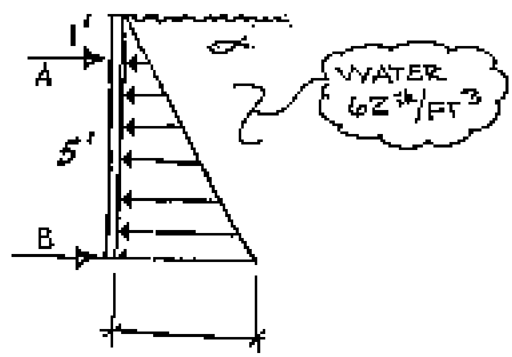
9VII95

Lecture 20
Example Problem

Triangular Loads

Given:
the distributed loads indicated below.
Determine:
the resultant load for each of the distributed loads.





Solution:
The total load for the problem at the left is determined by the area under the loading envelope. Since this load is an iscosoles triangle simply take 1/2 the base and multiply it by the maximum magnitude of the loading which is equal to the height;

1/2 (6 ft)(1200 #/ft) = 3600# down

Since the loading is symmetrical, the resultant will act along the centerline of the triagular loading, or 3 ft from A.

The hydrostatic problem on the right is a bit more complex. First note that water weighs 62.5 pcf and that deeper beneath the surface of water, the greater the pressure and loading on the indicated wall. The greatest magnitude of the load is taken as th e weight of water multiplied by the depth; 62.5 pcf x 5 feet = 375 psf at the base of the wall. The total load is then found in a manner similar to above. Multiply 1/2 (base)(greatest magnitude of the force).

1/2 (5ft)(375 #/ft^2) = 937.5#/ft of wall

Note that this equation gives the force acting on a piece of wall 5 ft high by 1 ft wide simply because of the nature of the weight of water being a volumetric measurement.

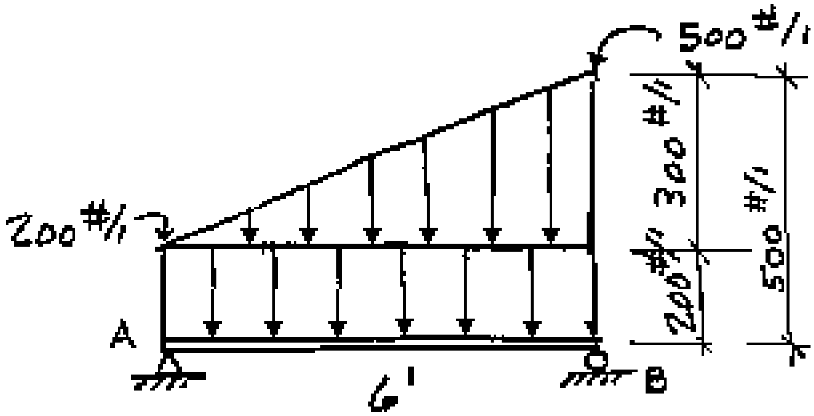
The resultant of the load will pass throught the cetroid of the area represented by the load. This is at a point 1/3 the distance from the greater side of the triangle and perpendicular to the member that is being loaded. In this case 5ft / 3 or 1.66 ft from the bottom.

Another way to determine this point is to draw a line from each point of the triangle to the midpoint of the opposite side. The point at which these lines intersect is also the centroid of that area.

Lecture 20
Example Problem

Trapezoidal Loads

Given: the trapezoidal distributed load indicated below.
Determine: the resultant load.



Solution: As indicated earlier, it is critical to recognize how a problem can be broken down into smaller problems that are more easily solved. This is a very simple case, and one in which the final result is easily determined in parts, but almost impossible if attempted as a whole.

Find the resultants of the two loads. First, the rectangular load;

(200#/ft)(6ft) = 1200# acting at midspan

Next, find the resultant of the triangular load

1/2 (6ft)(300#/ft) = 900#

This force acts at 2 feet from the right end of the beam.

They can now be combined and the reactions found. The total load is equal to the sum of the two smaller resultants or 1200# + 900# = 1200#. Find its location using sum M_A:

```
2100#(x) + B(6ft) = 1200#(3ft) + 900#(4ft) + B(6ft)
         2100#(x) = 3600#ft +3600#ft
            2100x = 7200#ft
                x = 3.43 ft from A
```

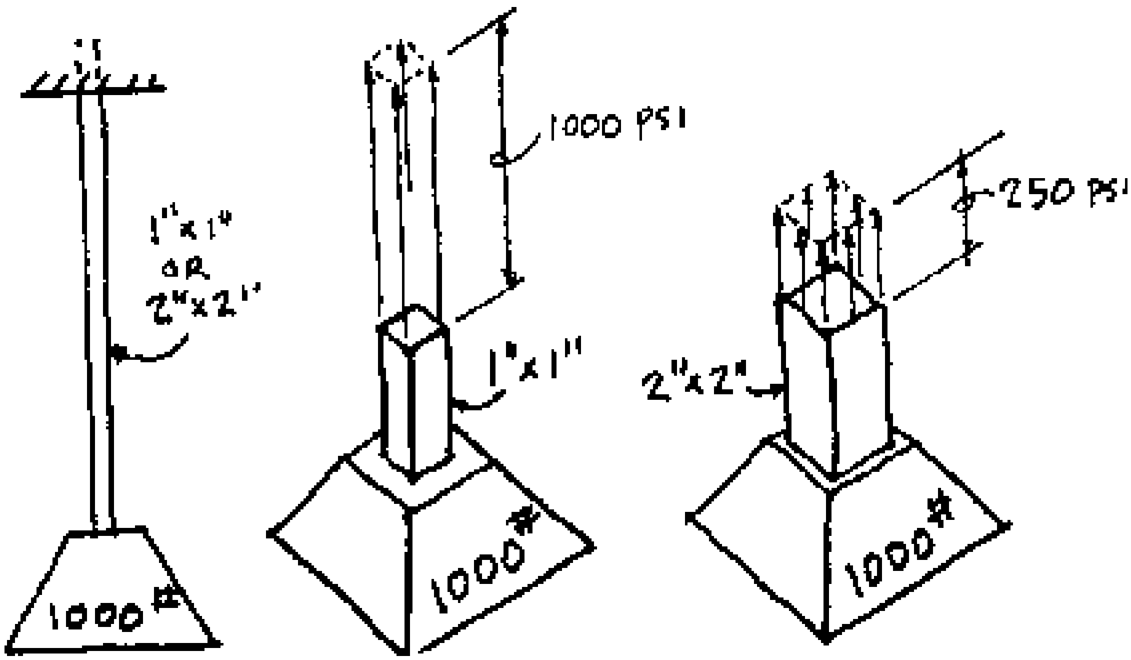
So, the resultant load equals 2100# acting 3.43 ft from A.

9VII95

Lecture 23
Example Problem

Stress/Area Relationship

Given:
a 1000 lb load suspended from a ceiling by a 1"x 1" member.
Determine:
a. the stress in the member
b. the stress if the size of the member is increased to 2" x 2".



Solution:
a. The internal force in that member is 1000# tension and the tension stress (intensity of the force per unit of area) is 1000 pounds divided by the area of 1 square inch. This is a stress of 1000 psi.

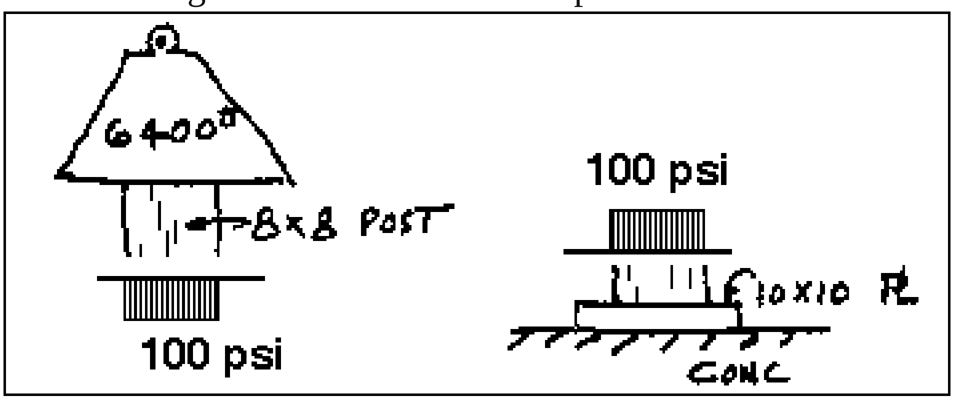
b. The 1000# load is now distributed evenly across an area of 4 square inches; thus, 1000# divided by 4 in^2 is a stress of 250 psi. This clearly demonstrates the inverse relationship between stress and area.

It becomes obvious that one way to reduce the total stress on a member is to increase its cross section. The other way would be to reduce the load.

Lecture 23
Example Problem

Compressive Stress

Given:
a 6400# weight is supported by an 8" x 8" post.
Determine:
a. the compression stress in the 8" x 8" post
b. the bearing stress between the post and the steel plate
c. the bearing stress between the steel plate and the concrete.



Solution:
a. The compressive force in the post is distributed over a cross-sectional area of 64 square inches: 6400 pounds/ 64 in^2 = 100 psi

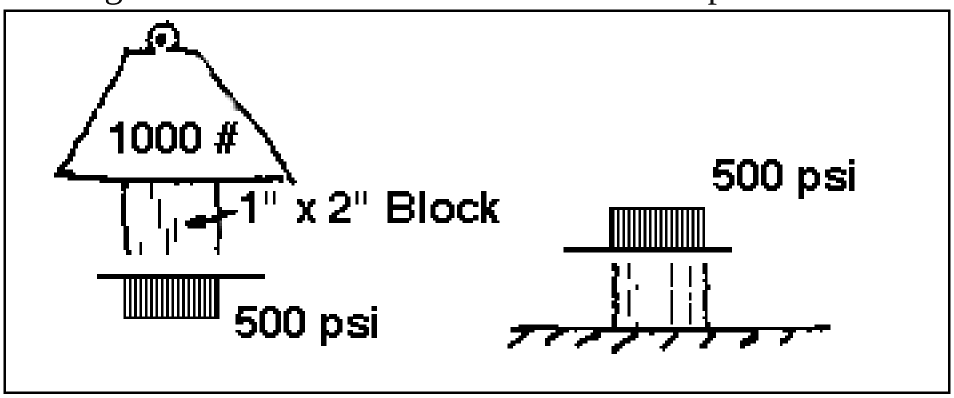
b. The stress at the point between the post and the steel bearing plate is dictated by the smallest area being loaded. The post is smaller than the plate, so it is the critical surface. The stress between the two is again 6400 pounds divided by 64 square inches for a total stress of 100 psi.

c. The stress between the plate and the concrete floor is also determined by the smallest area. In this case it is the 10" x 10" plate. The load remains the same 6400 pounds but it is now distributed over an area of 100 square inches. The resulting stress is 64 psi, a clear reduction in the stress.

Lecture 23
Example Problem

Drawing a Stress Prism

Given:
a 1" x 2" block is loaded with 1000#
Determine:
the magnitude of the stress and draw the stress prism



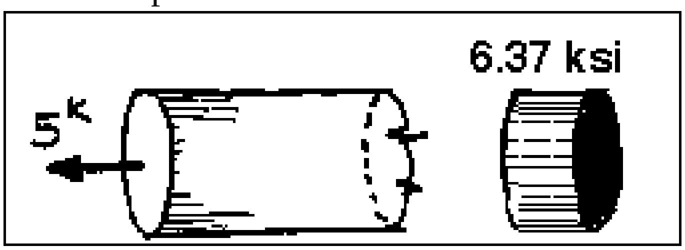
Solution: Knowing that the internal stress f = P/A , simply take the 1000 pounds and divide it by the area of the element. 1000# / 2 in^2 = 500 psi The stress is evenly distributed over the cross-section with a magnitude of 500 psi. If the volume of the stress prism is determined it can be seen that it is the same as the total load on the section. Volume = width x length x height; = 1 inch x 2 inches x 500 psi = 1000 pounds.

Note: The even distribution of this axial load is represented by a simple rectangular stress prism. If the load was moved to one side of the axis the stress prism would also have to change in response to the changing state of stress within the se ction. The prism would deepen on the side that the load was being applied and reduce in size on the opposite side. This would continue until the sign of the stress would actually change, indicating that part of the cross section was no longer in compres sion, but in tension to maintain equilibrium.

Lecture 23 Example Problem

More Stress Prisms

Given:
a 1" diameter rod loaded with a 5k force as indicated.
Determine:
the stress prism.



Solution:
The stress prism of the axial load would be evenly distributed over the cross-section. The magnitude of the stress (f) will equal the force divided by the area. In this case the cross section is circular so the area will equal:

$$A = \frac{1}{2}r^2 = 3.14(.5in)^2 = .785\ in^2$$

The stress will equal:

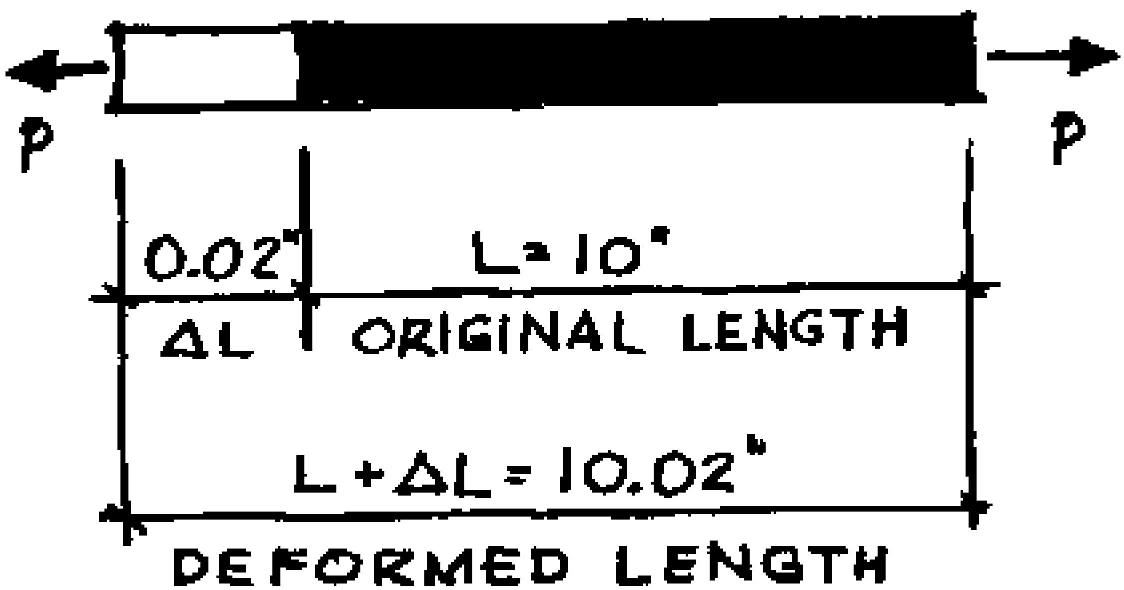
$$f = 5k / .785\ in^2 = 6.37\ ksi \text{ or } 6370\ psi$$

The stress prism will take the shape of a cylinder with the area or the cross section as its base and the magnitude of the stress as its height.

Lecture 25
Example Problem

Modulus of Elasticity

Given:
a 1" x 2" x 10" long aluminum specimen loaded with P = 40,000 pounds deformed a total 0.02 inchesas it was tested within the elastic range.
Determine:
a. the stress in the material
b. the strain in the material
c. the modulus of elasticity.



Solution:
a. Direct stress is the axial load divided by the cross-sectional area. This is

40 kips / 2in^2 = 20,000 psi.

b. The strain is the amount of deformation divided by the initial length. This is

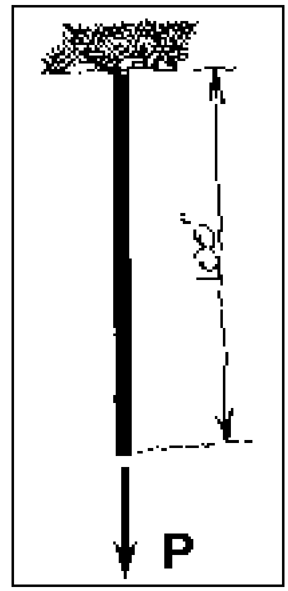
(0.02 in / 10 in = 0.002 inches per inch.

c. The Modulus of Elasticity is the stress divided by the strain.

20,000 psi / 0.002 in/in = 10,000,000 psi.

Lecture 25
Example Problem

Deformation



Given:
the 100 inch 1/2 inch diameter steel bar loaded in axial tension with the 10k load (P) as shown.
Determine:
the elongation of the bar.
Solution:
Knowing that the Modulus of Elasticity = stress/strain, and that strain is delta L/L, it is possible to solve for delta L. The Modulus of Elasticity of normal grades of steel is 29,000 ksi. Now, write out the full equation for the Modulus of Elasticity.

E = stress/strain

29,000 ksi = (P/A) / (delta L/L)

Because we are solving for delta L, rearrange the equation to isolate delta L.

delta L = (P)(L) / (A)(E)

Finally, substitute all the known values to solve for delta L;

delta L = (10k)(100in) / 0.196 in^2) (29,000 ksi)

delta L = 0.176 in

Lecture 27
Example Problem

Thermal Strain

Given:
A 4 innch diameter (a = 3.17 in^2), 25 ft long standard steel steam pipe is installed at 60 degrees F. The pipe temperature increases to 220 F when in use.
Determine:
the elongation of the pipe

Solution:
First, find the equation necessary to solve for delta L due to thermal expansion.

delta L = (alpha)(delta T)(L)

Find the coefficient of thermal expansion from the lecture table for steel (alpha = 0.000 0065 in per degree F) and the temperature variation (220 - 60 = 160 degrees F). Now replace the variables in the equation above to solve for delta L.

(0.000 0065 in/F)(160F)(25 ft)(12in/ft)

Delta L (elongation) = 0.312 in

Thus, the total deformation would be 0.312 inches and the overall length would be 300.312 inches.

Given:
assume in the above example that the pipe had been anchored "rigidly" in two "immovable" concrete buttresses, so that the pipe could not elongate.
Determine:
the stress in the pipe is caused by this temperature change.

Solution:
This now becomes a stress/strain problem rather than a thermal stress problem. The key to solving this problem is realizing that the stress caused by the fixed supports is effectively the same as a pipe with an

original length of 300.312 in that had been shortened to 25 feet or 300 inches. Start with E = stress / strain and solve for stress.

```
        stress = (E)(strain)
               = (E)(delta L/L)
               = (29,000,000 psi)(0.312 inches)/(300 inches)
               = 30,160 psi.
```

this exceeds the acceptable 24 000 psi limit.......

Determine:
What force would this exert on buttresses?

Solution:
Knowing that stess is equal to the force divided by the cross-sectional area, turn this around to get: Force = (stress)(area).

(30160 psi)(3.17in^2) = 95,600 poinds of force. (quite a large force!!!)

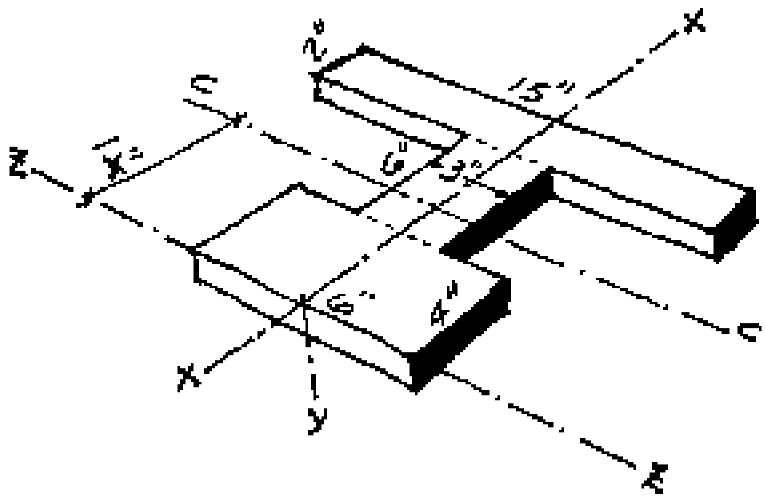
These problems should illustrate the incredible force that a variation in termperature can exert upon a structure.

Lecture 28

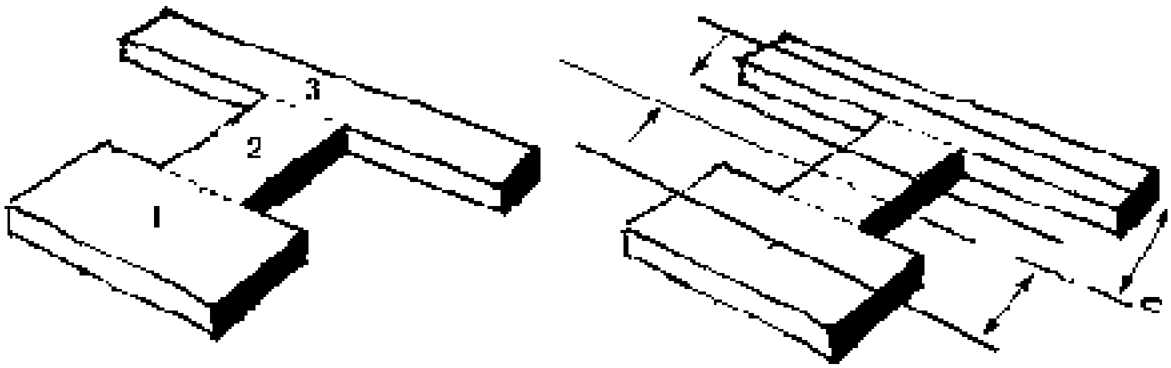
Example Problem

Center of Gravity

Given:
the plate shown in the diagram has a weight of 1.0#/in^2 of horizontal surface.
Determine:
the center of gravity of the plate knowing that it is symmetrical about the X-X axis.



Solution:
The principle of moments states that the total weight about an axis is equal to the sum of the moments of the component weights about that same axis. Thus, the first thing to do is to divide the plate into several simple parts. Then, determine the area and the center of gravity (or centroid) for each of the component parts. After this is completed, take the moments of each of the parts around a convenient axis (in this case select the Z-Z axis about which to take these moments).



$$\text{Sum } M_{A_{total}} = M_{A_1} + M_{A_2} + M_{A_3}$$

This simple equation can be rewritten as follows in which each of the component parts are described:

(A_{total})(distance from reference axis to centroidal axis) = (A_1)(distance from centroid of A_1 to reference axis) + (A_2)(distance from centroid of A_2 to reference axis) + (A_3)(distance from centroid of A_3 to reference axis)

[(6in)(4in)]**(2in)** + [(6in)(3in)]**(7in)** + [(15in)(2in)]**(11in)** = (24 +18+30)**(y)**

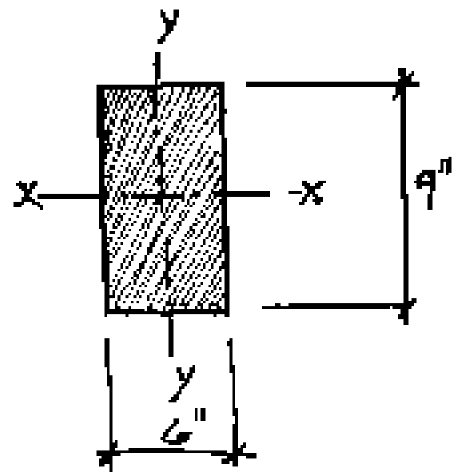
48 in^3 + 126 in^3 + 330 in^3 = 72 in^2 y
504 in^3 = 72 in^2 y

and then solving for y ... the centroidal axis is **7 inches** from the reference axis.

The actual center of gravity occurs midway through the depth of the plate at the point calculated above. As the plate thickness is reduced the line of action of the center of gravity will remain while the center of gravity moves proportionally along this line of action always effective at the midpoint of the depth of the plate. If the plate thickness is reduced to zero it has no weight and the former center of gravity position is now referred to as the centroid of the area.

Lecture 28
Example Problem

Moment of Inertia



Given: the cross-section to the right.
Determine: The Moments of Inertia, I_{xx} and I_{yy} of this section.

Solution:
The moment of inertia of a rectangular shape such as this one is easily calculated by using the equation $I = 1/12\ bh^3$. However, it is crucial that b and h are assigned correct values.

$I_{xx} = 1/12(6in)(9in)^3 = 364.5\ in^4$

$I_{yy} = 1/12(9in)(6in)^3 = 162\ in^4$

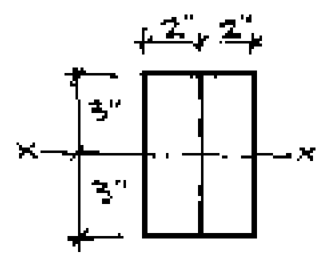
In this case, observation will confirm the choices for b and h. It is logical that I_{xx} is greater than I_{yy} because a larger amount of the rectangular area lies further from the x-x axis than the y-y axis. This causes the shape to have a greater resistance to rotation around the x-x axis and therefore a larger moment of inertia around that axis.

COURSEWARE
STRUCTURES I EXAMPLE PROBLEMS
BACK

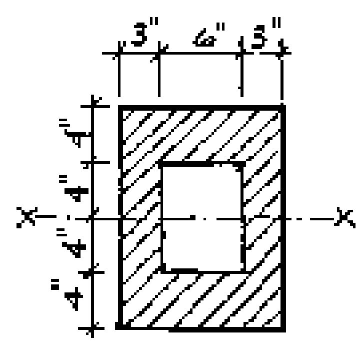
Lecture 28
Example Problem

Built-Up Sections

Given:
the following cross sections
Determine:
I_x of each section considering its component parts.



Solution:
The top shape can either be treated as two smaller rectangles or one large rectangle. It CANNOT be treated as four small rectangles using the rectangle formula because that formula gives the moment about the axis through the centroid of the area only. Therefore, the centroidal axis of the small areas must be the same as the centroidal axis of the larger area. Later a formula will be given which makes it possible to transfer moments from one centroidal axis to another, but that is not possible at this t ime.



I_{xx} as two small rectangles:

$$I_{xx} = (2)(1/12)(2in)(6in)^3 = \mathbf{72\ in^4}$$

I_{xx} as one large rectangle:

$$I_{xx} = 1/12(4in)(6in)^3 = \mathbf{72\ in^4}$$

The trick to the second shape is to find the moment of inertia of the large shape as if it were a solid rectangle and then subtract the moment of inertia of the smaller rectangular shape of the hole. The remainder will be the moment of inertia of the rin g. This is possible because the two shapes have the same centroidal axis.

I_{xx} = 1/12(12in)(16in)^3 - 1/12(6in)(8in)^3

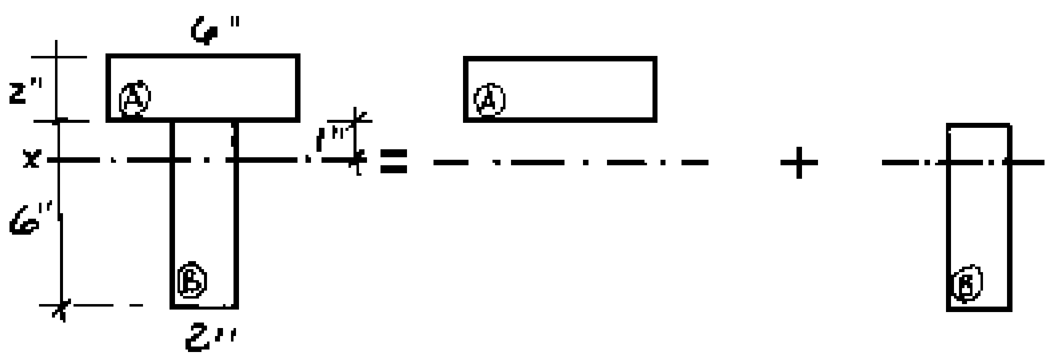
4096 in^4 - 256 in^4

3840 in^4

Lecture 28
Example Problem

Transfer Formula

Given:
the glued asymmetric built-up cross-section below.
Determine:
the moment of inertia of the composite area about the x axis.



Solution:
The transfer formula was invented for cases such as this where a composite shape requires a single moment of inertia and the individual parts do not share their centroidal axis.

I_{xx} = sum ($I_c + Ad^2$)

In this case:
$I_{xx} = I_A + A_A d^2 + I_B + A_B d^2$

$I_{xx} = 1/12(6in)(2in)^3 + (12in^2)(2in)^2 + 1/12(2in)(6in)^3 + 12in^2(2in)^2$

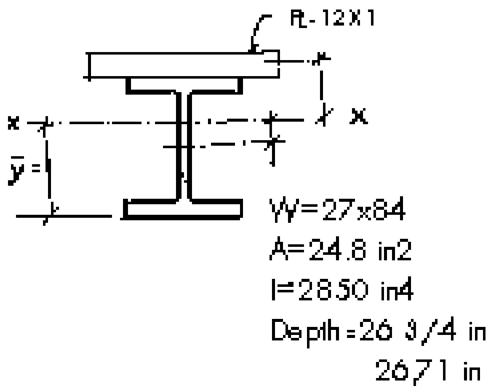
$I_{xx} = 4\ in^4 + 48\ in^4 + 36\ in^4 + 48\ in^4$

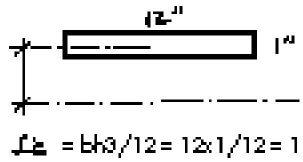
$I_{xx} = 134\ in^4$

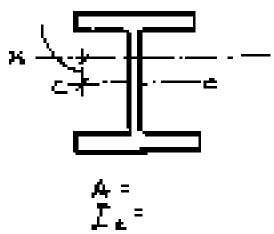
Lecture 28
Example Problem

Centroidal Axis & the Transfer Formula

Given:
a built-up bridge girder section.
Determine:
its centroidal axis x-x and I_{xx}.







SECTION	A	y	Ay	I_c	d	Ad^2	I_c+Ad^2
(plate)							
(I-beam)							
Σ		/		/	/		

Solution:
For this problem, before the combined Moment of Inertia can be calculated, the centroidal axis, x-x must be found. When a Moment of Inertia problem gets complex, it can be very useful to utilize a chart such as the one illustrated above to keep track of the many bits of information necessary to solve. As the problem is worked below, fill in the chart given in the above illustration.

Begin by filling in known quantites. In this case, A and I_c for both pieces. The values for the wide flange beam are given in the illustration (A = 24.8 in^2; I_c = 2850 in^4) and those for the bar are easily calcu lated (A = 12 in^2; I_c = 1/12(12in)(1in)^3 = 1 in^4). The total area equals 24.8 in^2 + 12 in^2 = 36.8 in ^2.

Next find 'y' for each piece. 'y' will be the distance between the centroid of each area and a common axis. Use the bottom edge of the wide flange section. For the purposes of this explaination, '1' will signify the bar and '2' will signify the wide fl ange section. Therefore, y_1 will equal the total height of the wide flange section plus half the height of the bar or

y_1 = 26.75in + .5in = 27.25in

y_2 will equal half the depth of the wide flange section or

y_2 = 26.75in/2 = 13.375in

Now Ay can be calculated for each part as well as the whole.

A_1y_1 = 12in^2 (27.25in) = 327 in^3
A_2y_2 = 24.8in^2 (13.375in) = 331.7 in^3
A_Ty_T = 327 in^3 + 331.7 in^3 = 658.7 in^3

From this, y_T, the centroidal axis can be found by dividing A_Ty_T by A_T or

658.7 in^3/36.8 in^2 = 17.9 in

This means that y_T is 17.9in away from the previous common axis which was the bottom of the wide flange beam.

Now the centroidal axis can be used to calculate the final variable, d, the absolute distance between the centroidal axis of the whole and the centroidal axis of each of the parts.

d_1 = 27.5in - 17.9in = 9.35 in
d_2 = 13.375in - 17.9in = 4.525 in

Next, calculate Ad^2 and apply the transfer formula to find I_{xx}.

I_{xx} = Ic1 + A1d1^2 + Ic2 + A2d2^2
I_{xx} = 1 in^4 + 12in^2 (9.35in)^2 +2850 in^4 + 24.8in^2 (4.525in)^2
I_{xx} = 1050 in^4 + 3358 in^4
I_{xx} = 4408 in^4

In conclusion, the centroidal axis, x-x, is located **17.9in** above the bottom of the wide flange beam, and the moment of inertia of the composite area equals **4408 in^4**.

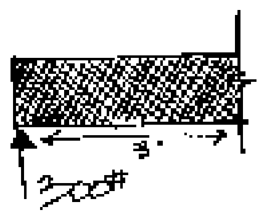
STRUCTURES I EXAMPLE PROBLEMS

BACK

Lecture 33
Example Problem

Internal Forces

Given:
the left cut section shown below.
Determine:
all forces acting at the cut section using the equations of equilibrium



Solution:
The shearing force applied to the end of the cut section is exactly equal to the internal shearing force before we cut the beam. It is numerically equal (but opposite in sense) to the algebraic sum of the loads and reactions on the FBD to the left of the cut. Thus it has the value of 300# upwards. We can see then, that this creates a couple which clearly puts the segment out of equilibrium.

By the sum M = 0, we can then find the moment required to put the system back into equilibrium. Remember, the magnitude of a couple is equal to the magnitude of one force multiplied by the distance between the forces.

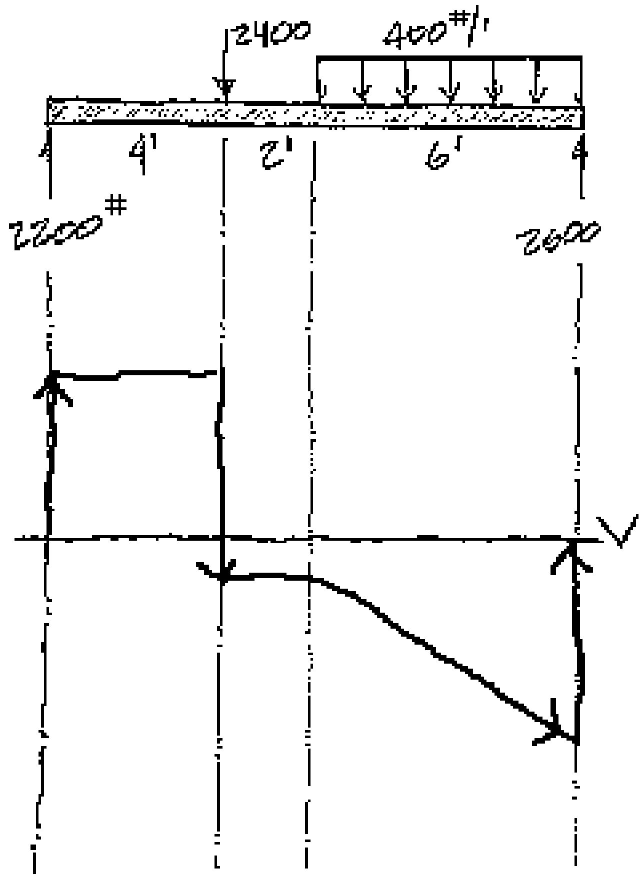
(300#)(3ft) + M = 0
M = 900 #ft counterclockwise

The moment that has been applied to the cut section of the FBD is the internal or resisting moment. Is it obvious that it is numerically equal to, but opposite in direction, to the moment caused by the loads and reactions?

Lecture 34
Example Problem

Shear Diagrams

Given:
the "weightless" beam and loading condition shown.
Determine:
the shear diagram.



Solution:
Construct the shear diagram by finding the sum F_y at strategic points along the length of the beam from the left end to the right. First, cut a section immediately to the right of the left reaction, which will give a shear equal to th e reaction force of 2200#. The force is posative, so the plot will lie above the x axis.

Continue to move along the beam length plotting the change in load; there is no change until the 2400# load, at which the shear drops 2400# from 2200# yielding a shear of -200#.

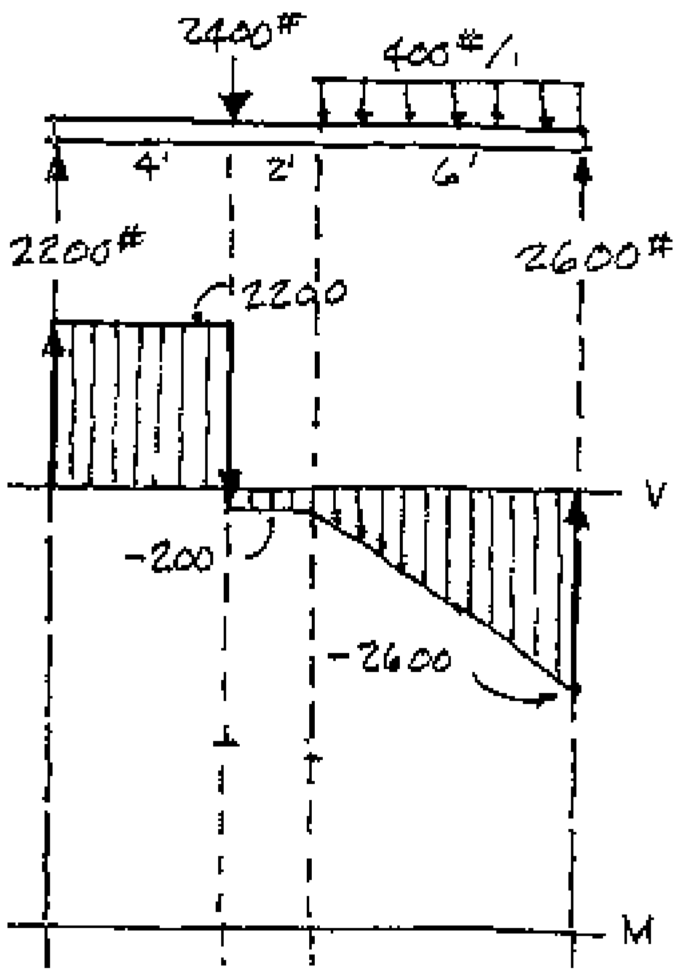
Again, the shear remains constant until the distributed load. At that point, it drops 400# for each foot, (or 100# for each 1/4 foot of length, etc.), with a total drop of (400#)(6ft) or 2400#. The value of the shear diagram reaches a low of -2600# at t he end of the beam.

As soon as the magnitude of the reaction at the right support is included, the diagram should close so that the shear returns to zero. The right reaction equals 2600# so the diagram does indeed close to zero.

Lecture 35
Example Problem

Moment Diagrams

Given:
the same beam and loading as in the shear diagram example.
Determine:
the moment diagram using the area method.



Solution:
The moment diagram will be constructed by once again moving along the beam from left to right and utilizing the values of the shear diagram.

At the left end of the beam the moment will equal zero. The 220# force will cause the moment to increase at a constant rate of 2200#/ft. The value of the moment diagram at the point four feet from the left end can be found using the area method. Its va lue will be equal to the area of the shear diagram to that point or (2200#)(4ft) = 8800#ft.

Continuing to the right, the shear diagram drops to -200#. The shear diagram is constant, so the moment diagram will once again have a constant slope. This time the slope will be negative because the shear diagram is negative. Using the area method, th e value of the moment diagram at 6 ft will be 8800#ft - (200#)(2ft) = 8400#ft.

From here, the moment diagram must close to zero because no concetrated moment acts on the right end. The shear diagram for this portion has a constant negative slope with an increasing ordinate. therfore, the moment diagram will follow an x^2 curve wit h a slope approaching the vertical.

The closure of the moment diagram can and should be checked using the area method. The area of this part of the shear diagram is a composite area consisting of a rectangle and a triangle. The shear diagram is also negative, so the area will be subtracte d from the last know moment value of 8400#ft.

8400#ft - (200#)(6ft) - 1/2(400#/ft)(6ft)(6ft) = 8400#ft - 1200#ft -7200#ft = 0

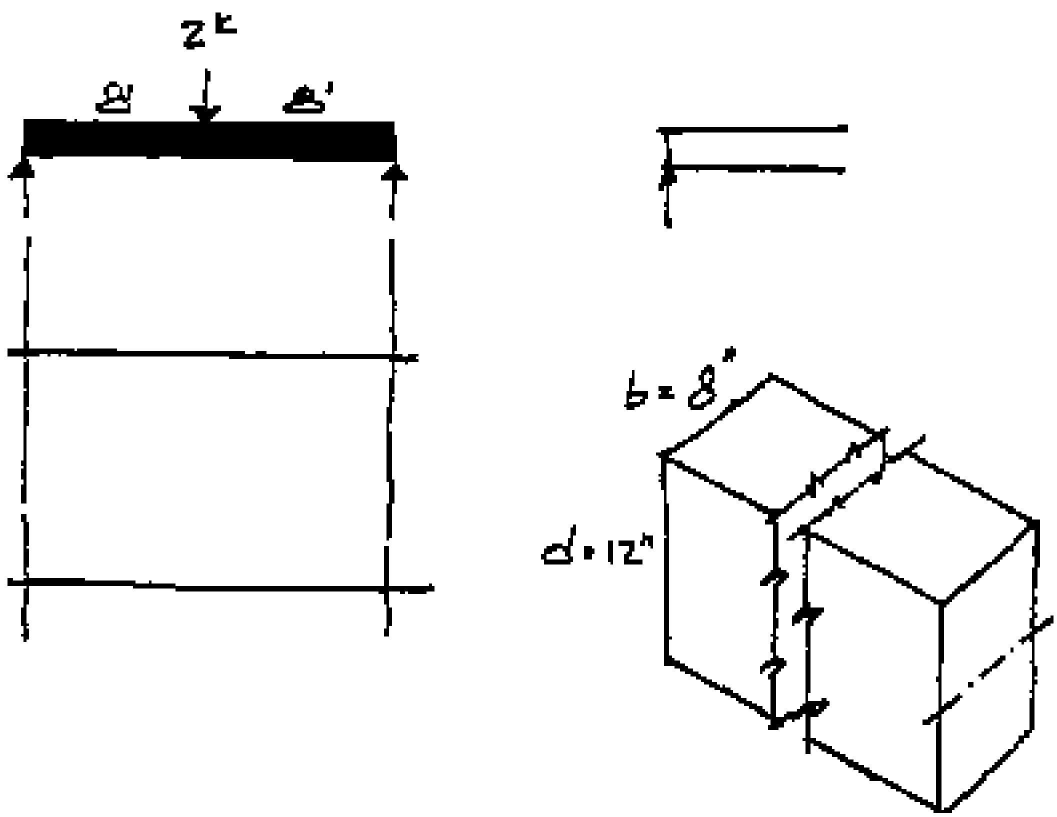
The moment diagram will close to zero.

Note that a beam such as this can experience posative and negative shear and still maintain a moment which remains posative along the entire length of the beam!

Lecture 38
Example Problem

Bending Stress in a Beam

Given:
the rough wood beam spanning 16ft and loaded with a concentrated load of 2000# as shown below. The beam is 12 in deep and 8 in wide.
Determine:
the maximum bending stress in this beam.



Solution:
First find the reactions and draw the shear and moment diagrams. From the moment diagram it is obvious that the maximum internal moment occurs 8 ft from the left support and is equal to 8000#ft.

To find the maximum bending stress, use the formula
f = Mc / I.
M = 8000#ft
c = 6 in
I = 1/12 bh^3 = 1152 in^4.

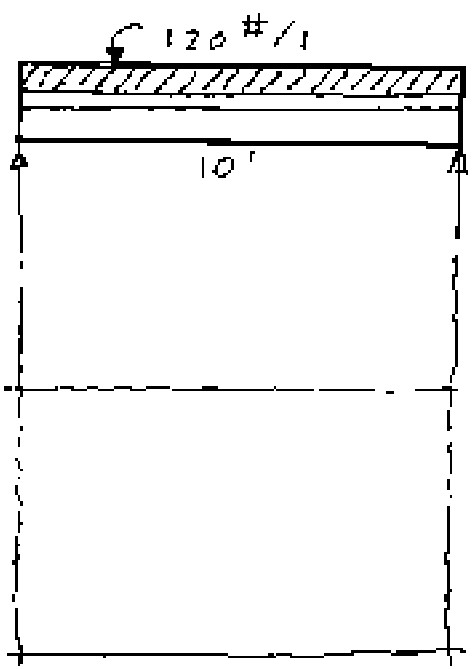
Therefore:
f = (8000#ft)(12 in/ft)(6 in) / 1152 in^4 = 500 psi

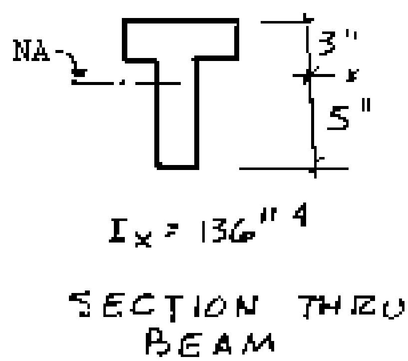
Lecture 38
Example Problem

Bending Stresses

Given:
the T-beam of the indicated section spans 10 ft and is loaded with a distributed load of 120#/ft as shown below.

Determine:
a. the maximum bending stress in tension and compression in this beam;
b. the maximum bending stresses 3 ft from the left end;
c. the bending stress in the beam 3 ft from the left end and 2 in below the neutral axis.





Solution:
First draw the shear and moment diagrams. The moment diagram shows that the maximum internal bending moment will occur 5ft from the left end and will equal 1500#ft.

In each case use the formula f = Mc/I

a. Max tension:
M = 1500#ft
c = 5 in
I = 136 in^4
f = (1500 #ft)(12 in/ft)(5 in) / 136 in^4 = **660 psi**

Max compression:
M = 1500#ft
c = 3 in
I = 136 in^4
f = (1500 #ft)(12 in/ft)(3 in) / 136 in^4 = **396 psi**

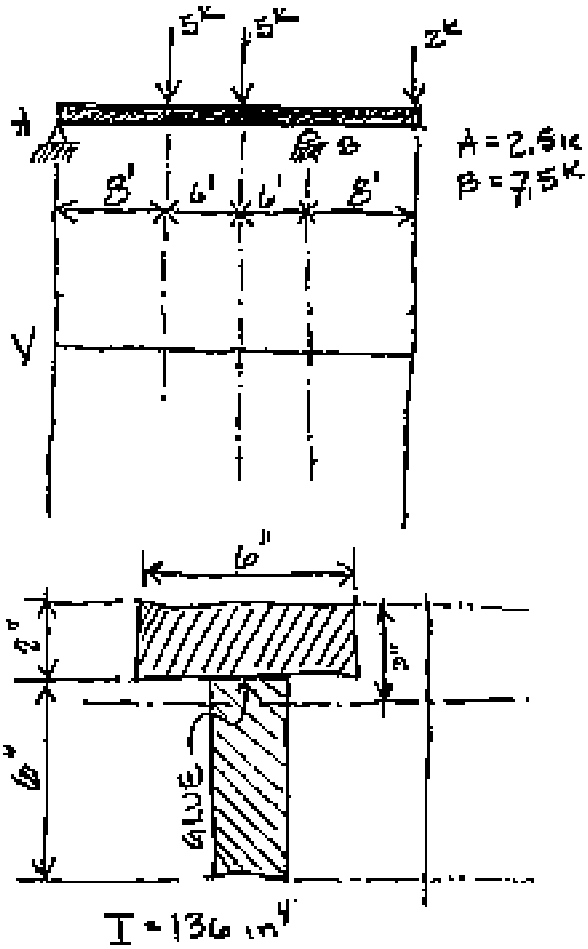
b. bending stresses at 3 ft
M at 3' = 600#(3ft) - 120#/ft(3 ft)(3/2 ft) = 1260 #ft
Tension:
f = (1260 #ft)(12 in/ft)(5 in) / 136 in^4 = **552 psi**
Compression:
f = (1260 #ft)(12 in/ft)(3 in) / 136 in^4 = **336 psi**

c. bending stress 2 in below the neutral axis
f = My/I = (1260 #ft)(12 in/ft)(2in) / 136 in^4 = **222 psi tension**

Lecture 39
Example Problem

Shear Stress

Given:
the beam consisting of two 2" x 6" pieces of plastic glued together subjected to the loading shown (downward forces from left to right are 5k, 3k and 2k).
Determine:
a. the maximum horizontal shearing force in the beam at max V
b. the magnitude of the shearing stress the glue must develop



Solution:

First plot the shear diagram for the beam.

a. From the shear diagram, the maximum shear force occurs at 14 ft from the left end and is equal to -5.5 k.

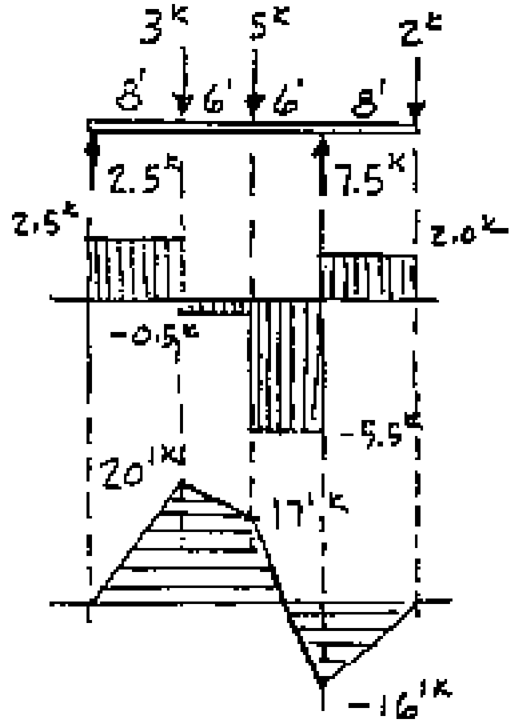
b. the shearing stress in the glue
f_v = VA'y / Ib
V = max shear force = 5.5k
A' = area above glueline = (6in)(2in) = 12 in^2
y = distance from NA to centroid of A' = 2 in
I = moment of inertia = 136 in^4
b = width of glueline = 2 in

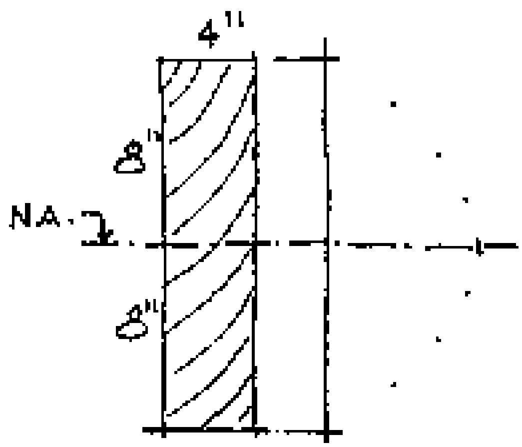
f = (5.5k)(12 in^2)(2in) / (136 in^4)(2in) = **.48 ksi**

Lecture 39
Example Problem

Plotting Shearing Stresses

Given:
The beam with loading condition shown below
Determine:
a. the horizontal shearing stress at 2in increments from top to bottom of the beam
b. plot these stresses - use maximum vertical shear on the beam





Solution:

From the shear diagram, the maximum shear force equals 5.5k = 5500#.

I = 1/12(4in)(16in)^3 = 1365 in^4

a. At each increment, from top to bottom, use the formula **fv = VA'y/Ib**

at 2":

fv = (5500#)(8in^2)(7in)/(1365 in^4)(4in) = **56.4 psi**

at 4":

fv = (5500#)(16in^2)(6in)/(1365 in^4)(4in) = **96.7 psi**

at 6":

fv = (5500#)(24in^2)(5in)/(1365 in^4)(4in) = **120.9 psi**

at 8":

fv = (5500#)(32in^2)(4in)/(1365 in^4)(4in) = **128.9 psi**

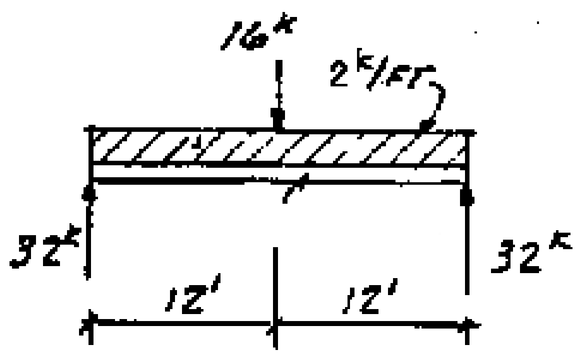
The lower half of the beam will reflect the numbers on the upper half with the shearing stress decreasing at an increasing rate closer to the bottom.

b. The plot of the shearing stresses will follow a curve with the maximum shearing stress occuring at the neutral axis.

Lecture 40
Example Problem

Beam Deflection

Given:
the 24' long simply supported beam with a 2k/ft load distributed over it's entire length and a 16k concentrated load at the midspan. The distributed load includes the self weight of the W21x62 beam.
E = 29,000 ksi
I = 1330 in^4
Determine:
the actual deflection of the beam and compare it with an allowable deflection of 1/360th of the span (l/360).



Solution:
The loading is a combination of a distributed load and a concentrated load. The deflection for each of these loading cases should be found independently and then combined to give a total deflection. Be careful with units since not all given information has consistent unit designation.

First determine the deflection based solely on the distributed load.

```
delta = 5wL^4/384 EI
      = (5)(2kip/ft)(24^4ft)(12in/ft)^3/(384)(29,000ksi)(1330in^4)
      = .387 in
```

Second, determine the deflection based solely on the concentrated load:

```
delta = PL^3/48 EI
      = (16kips)(24^3ft)(12in/ft)^3/(48)(29,000ksi)(1330in^4)
```

= .206 in

Add these two deflections to determine the total deflection:

.387in + .206in = **0.593in**

compare this to the allowable deflection:

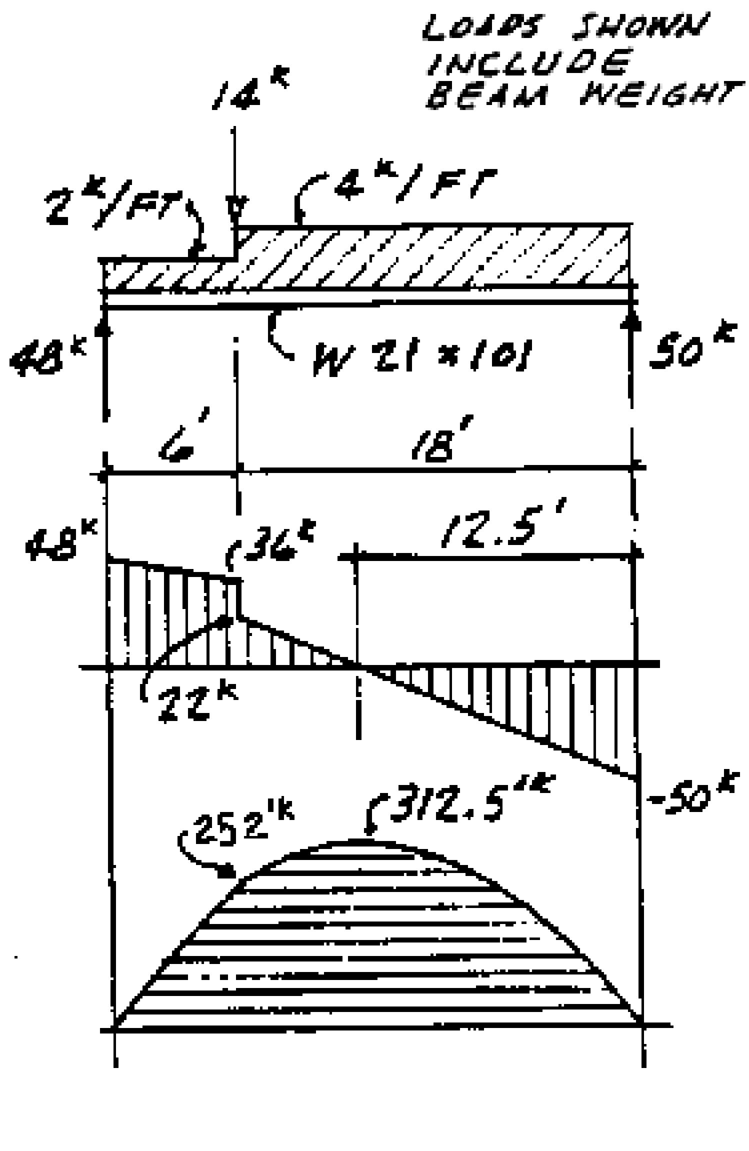
span/360 = (24ft)(12in/ft)/360 = **0.8 in**

0.593in < 0.8in - OK

Lecture 40
Example Problem

Deflection Approximation

Given:
the beam with the loading shown below.
Determine:
the deflection of the beam using an approximate method and compare it with an allowable deflection of 1/360th of the span.

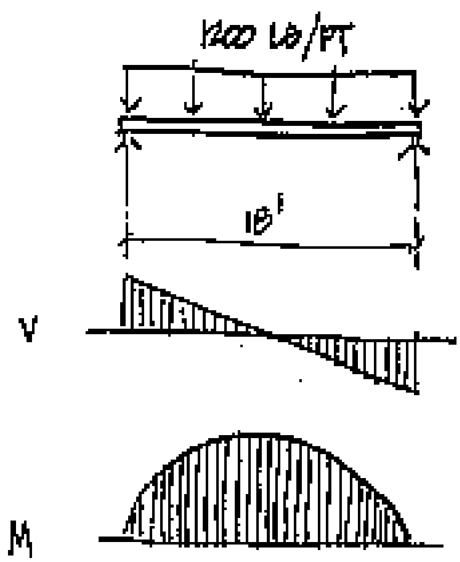


Solution:

Lecture 41
Example Problem

Beam Sizing #1

Given: the uniformly loaded steel beam shown
Determine: the most efficient W section for this loading based on bending and deflection. (Fb=24 ksi, Fv=14.5 ksi) (ignore the self-weight of the beam)



$W = w(18) =$

$V = \frac{W}{2} =$

$M =$

$S_{REQ'D} = \frac{M}{F_b} = \frac{\qquad}{24\ ksi} =$

SELECT

(CHECK SHEAR)

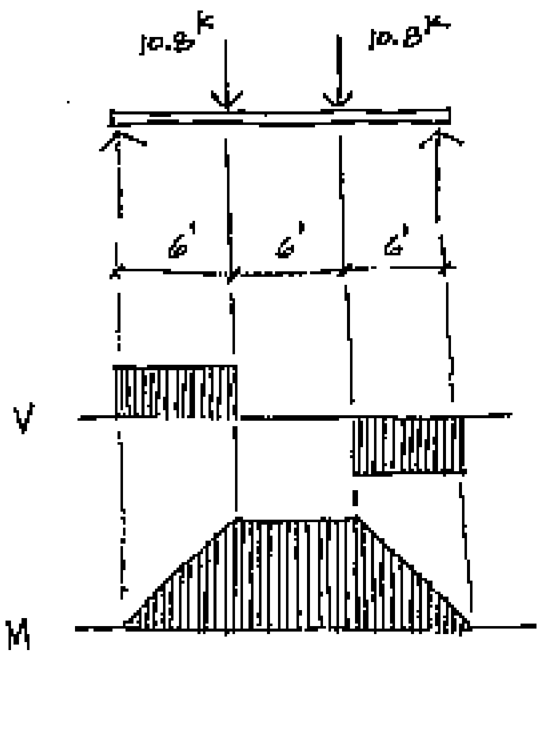
$\Delta_{LL} = \frac{18\ FT\ (12\ in)}{\qquad} =$

$\Delta_{ACT} = \frac{5WL^3}{384EI} = \frac{5}{384(29 \times 10^6)(\qquad)} =$

Lecture 41
Example Problem

Beam Sizing #2

Given:
the beam shown below with two concentrated loads of equal spacing.
Determine:
the most efficient W section for this loading based on bending, deflection, and shear. (Fb=24 ksi, Fv=14.5 ksi) (ignore the self-weight of the beam)(max D= l/360)



$V = R =$

$M =$

$S_{REQ'D} = \frac{M}{F_b} =$

SELECT

(CHECK SHEAR)

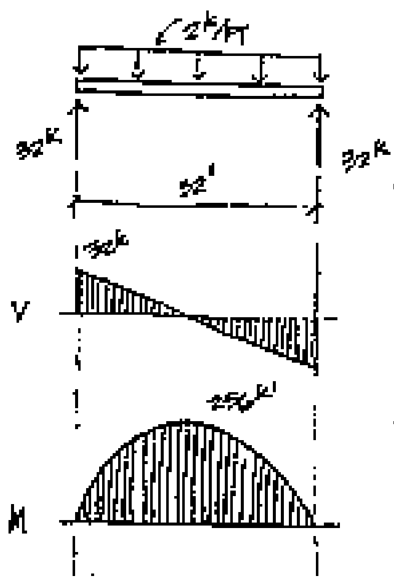
$\Delta_{ALL} =$ ______

$\Delta_{ACT} = \frac{Pa}{24EI}(3L^2 - 4a^2) =$

Lecture 41
Example Problem

Beam Sizing # 3

Given:
the uniformly loaded beam shown below.
Determine:
the most efficient W section for this loading based on bending, shear, and deflection. (Fb=24 ksi, Fv=14.5 ksi) (ignore the self-weight of the beam)(max D= l/360)



Solution:
First size the beam according to bending. The moment that the applied loads create is used and thus,

S= M/F = (256,000 #ft)(12 in/ft)/24,000psi = 128 in^3

this is the required Section Modulus. The steel tables are consulted and a section is chosen (W21x68 which has an S = 140 in^3). The required section must now be modified to reflect the fact that we have added the weight of the beam to the loads that th e beam must carry.

(actual load / assumed load)(S) equals

(2068 #/ft / 2000 #/ft)(128 in^3) = 132.5 in^3 < 140 in^3 so the beam is still OK.

Now check the shear (including the beam weight)

And now check the deflection of the beam.

Lecture 41
Example Problem

Section Efficiency

Given: the uniformly loaded beam shown
Determine: the most efficient sections for each material. (ignore the self-weight of the beam)(max D= l/240)

Lecture 41
