EN3: Introduction to Engineering and Statics

Division of Engineering
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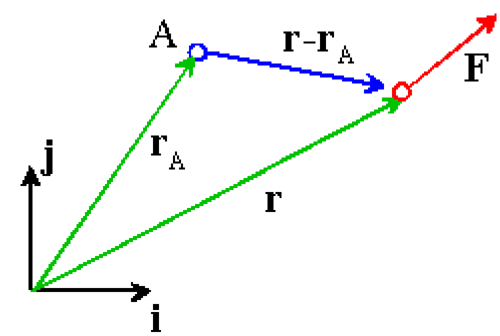
4. Moment of a force

The *moment* of a force is a measure of its tendency to rotate an object about some point. The physical significance of a moment will be discussed later. We begin by stating the mathematical definition of the moment of a force about a point.



4.1 Definition of the moment of a force.

To calculate the moment of a force about some point, we need to know three things:

1. The force vector, expressed as components in a basis (F_x, F_y, F_z), or better as $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$
2. The position vector (relative to some convenient origin) of the point where the force is acting (x, y, z) or better $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
3. The position vector of the point (say point *A*) we wish to take moments about (you must use the same origin as for 2) $(x^{(A)}, y^{(A)}, z^{(A)})$ or $\mathbf{r}_A = x^{(A)}\mathbf{i} + y^{(A)}\mathbf{j} + z^{(A)}\mathbf{k}$

The **moment of F** about point A is then defined as

$$\mathbf{M}_A = (\mathbf{r} - \mathbf{r}_A) \times \mathbf{F}$$

We can write out the formula for the components of $\mathbf{M}_A$ in longhand by using the definition of a cross product

$$\begin{aligned}\mathbf{M}_A &= [(x-x_A)\mathbf{i}+(y-y_A)\mathbf{j}+(z-z_A)\mathbf{k}]\times[F_x\mathbf{i}+F_y\mathbf{j}+F_z\mathbf{k}]\\ &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ (x-x_A) & (y-y_A) & (z-z_A)\\ F_x & F_y & F_z\end{vmatrix}\\ &= \left\{(y-y_A)F_z-(z-z_A)F_y\right\}\mathbf{i}+\left\{(z-z_A)F_x-(x-x_A)F_z\right\}\mathbf{j}+\left\{(x-x_A)F_y-(y-y_A)F_x\right\}\mathbf{k}\end{aligned}$$

The **moment of F about the origin** is a bit simpler

$$\mathbf{M}_O = \mathbf{r}\times\mathbf{F}$$

or, in terms of components

$$\begin{aligned}\mathbf{M}_O &= [x\mathbf{i}+y\mathbf{j}+z\mathbf{k}]\times[F_x\mathbf{i}+F_y\mathbf{j}+F_z\mathbf{k}]\\ &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ x & y & z\\ F_x & F_y & F_z\end{vmatrix}\\ &= \left\{yF_z-zF_y\right\}\mathbf{i}+\left\{zF_x-xF_z\right\}\mathbf{j}+\left\{xF_y-yF_x\right\}\mathbf{k}\end{aligned}$$

Often the best way to find the moment of a force about some point is simply to use the point of interest as the origin. Then you use the simpler formula.

4.2 Resultant moment exerted by a force system.

Suppose that N forces $\mathbf{F}^{(1)},\mathbf{F}^{(2)}...\mathbf{F}^{(N)}$ act at positions $\mathbf{r}^{(1)},\mathbf{r}^{(2)},...\mathbf{r}^{(N)}$. The *resultant moment* of the force system is simply the sum of the moments exerted by the forces. You can calculate the resultant moment by first calculating the moment of each force, and then adding all the moments together (using vector sums).

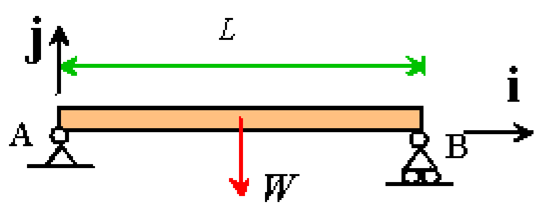
Just one word of caution is in order here – *when you compute the resultant moment, you must take moments about the same point for every force.*

Taking moments about a different point for each force and adding the result is meaningless!

4.3 Examples of moment calculations using the vector formulas

We work through a few examples of moment calculations

Example 1: The beam shown below is uniform and has weight W. Calculate the moment exerted by the gravitational force about points A and B.



We know (from the table provided earlier) that the center of gravity is half-way along the beam. The force (as a vector) is

$$\mathbf{F} = -W\mathbf{j}$$

To calculate the moment about A, we take the origin at A. The position vector of the force relative to

A is

$$\mathbf{r} = (L/2)\mathbf{i}$$

The moment about A therefore

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F} = (L/2)\mathbf{i} \times (-W)\mathbf{j} = -(WL/2)\mathbf{k}$$

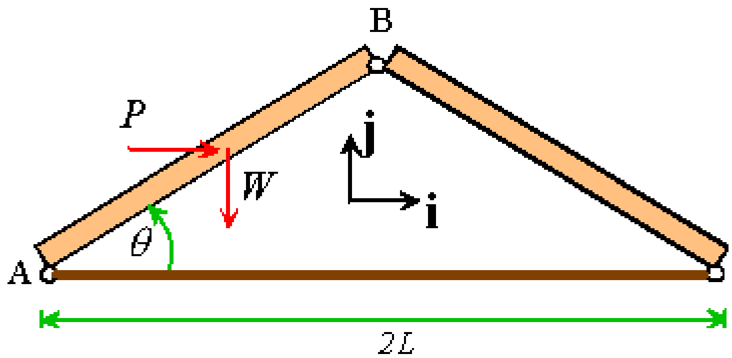
To calculate the moment about B, we take B as the origin. The position vector of the force relative to B is

$$\mathbf{r} = -(L/2)\mathbf{i}$$

Therefore

$$\mathbf{M}_B = \mathbf{r} \times \mathbf{F} = (-L/2)\mathbf{i} \times (-W)\mathbf{j} = (WL/2)\mathbf{k}$$

Example 2. Member AB of a roof-truss is subjected to a vertical gravitational force *W* and a horizontal wind load *P*. Calculate the moment of the resultant force about B.



Both the wind load and weight act at the center of gravity. Geometry shows that the position vector of the CG with respect to B is

$$\mathbf{r} = (-L/2)\mathbf{i} - (L/2)\tan\theta\mathbf{j}$$

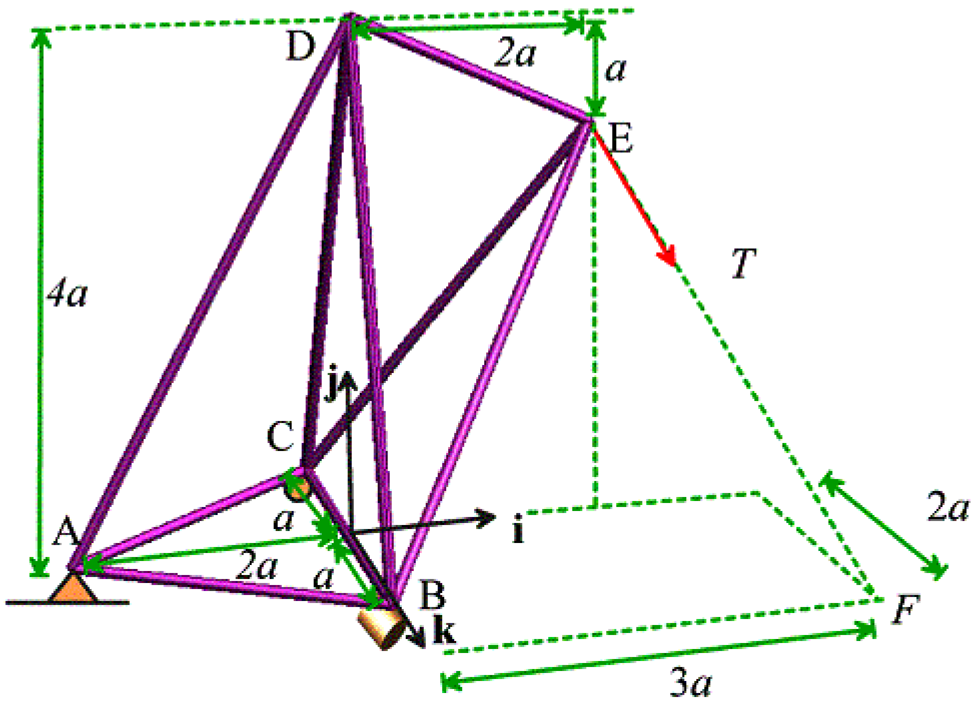
The resultant force is

$$\mathbf{F} = P\mathbf{i} - W\mathbf{j}$$

Therefore the moment about B is

$$\begin{aligned}\mathbf{M}_B &= \mathbf{r} \times \mathbf{F} = \left[(-L/2)\mathbf{i} - (L/2)\tan\theta\mathbf{j}\right] \times \left[P\mathbf{i} - W\mathbf{j}\right] \\ &= (L/2)\left\{W + P\tan\theta\right\}\mathbf{k}\end{aligned}$$

Example 3. The structure shown is subjected to a force *T* acting at *E* along the line *EF*. Calculate the moment of *T* about points A and D.



This example requires a lot more work. First we need to write down the force as a vector. We know the magnitude of the force is T, so we only need to work out its direction. Since the force acts along EF, the direction must be a unit vector pointing along EF. It's not hard to see that the vector EF is

$$\overrightarrow{EF} = a\mathbf{i} - 3a\mathbf{j} + 2a\mathbf{k}$$

We can divide by the length of EF ($a\sqrt{14}$) to find a unit vector pointing in the correct direction

$$\mathbf{e}_{EF} = (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})/\sqrt{14}$$

The force vector is

$$\mathbf{F} = T(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})/\sqrt{14}$$

Next, we need to write down the necessary position vectors

Force: $\mathbf{r} = 2a\mathbf{i} + 3a\mathbf{j}$

Point A: $\mathbf{r}_A = -2a\mathbf{i}$

Point D: $\mathbf{r}_D = 4a\mathbf{j}$

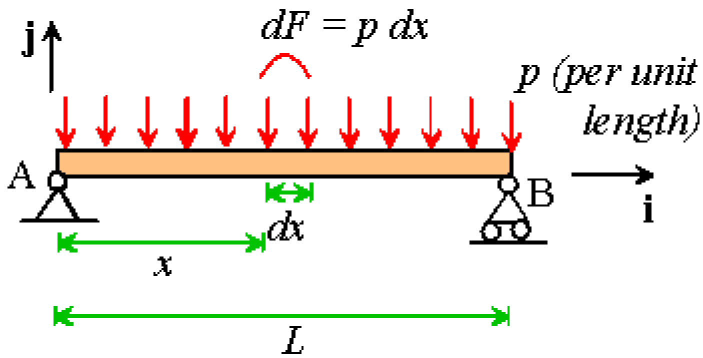
Finally, we can work through the necessary cross products

$$\begin{aligned}
\mathbf{M}_A &= (\mathbf{r} - \mathbf{r}_A) \times \mathbf{F} \\
&= (4a\mathbf{i} + 3a\mathbf{j}) \times T(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})/\sqrt{14} \\
&= \left(Ta/\sqrt{14}\right) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 0 \\ 1 & -3 & 2 \end{vmatrix} \\
&= Ta\left(6\mathbf{i} - 8\mathbf{j} - 15\mathbf{k}\right)/\sqrt{14}
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}_D &= (\mathbf{r} - \mathbf{r}_D) \times \mathbf{F} \\
&= (2a\mathbf{i} - a\mathbf{j}) \times T(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})/\sqrt{14} \\
&= \left(Ta/\sqrt{14}\right) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 0 \\ 1 & -3 & 2 \end{vmatrix} \\
&= Ta\left(-2\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}\right)/\sqrt{14}
\end{aligned}$$

Clearly, vector notation is very helpful when solving 3D problems!

Example 4. Finally, we work through a simple problem involving *distributed* loading. Calculate expressions for the moments exerted by the pressure acting on the beam about points A and B.



An arbitrary strip of the beam with length *dx* is subjected to a force

$$d\mathbf{F} = -p\,dx\;\mathbf{j}$$

The position vector of the strip relative to A is

$$\mathbf{r} = x\mathbf{i}$$

The force acting on the strip therefore exerts a moment

$$d\mathbf{M}_A = x\mathbf{i} \times (-p\,dx)\,\mathbf{j} = -\,px\;dx\;\mathbf{k}$$

The total moment follows by summing (integrating) the forces over the entire length of the beam

$$\mathbf{M}_A = \int_0^L -p\,x dx\;\mathbf{k} = -(pL^2/2)\mathbf{k}$$

The position vector of the strip relative to B is

$$\mathbf{r} = -(L-x)\mathbf{i}$$

The force acting on the strip exerts a moment

$$d\mathbf{M}_B = -(L-x)\mathbf{i} \times (-p\,dx)\,\mathbf{j} = p(L-x)\;dx\;\mathbf{k}$$

The total moment follows by summing (integrating) the forces over the entire length of the beam

$$\mathbf{M}_A = \int_0^L p(L-x)dx\;\mathbf{k} = (pL^2/2)\mathbf{k}$$

4.4 The Physical Significance of a Moment

A force acting on a solid object has two effects: (i) it tends to accelerate the object (making the object's center of mass move); and (ii) it tends to cause the object to *rotate*.

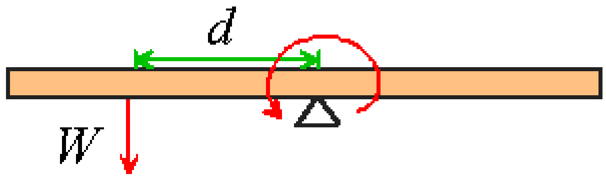
1. *The moment of a force about some point quantifies its tendency to rotate an object about that point*.

2. *The magnitude of the moment specifies the magnitude of the rotational force.*

3. *The direction of a moment specifies the axis of rotation associated with the rotational force, following the right hand screw convention.*

Let's explore these statements in more detail.

The best way to understand the physical significance of a moment is to think about the simple experiments you did with levers & weights back in kindergarten. Consider a beam that's pivoted about some point (e.g. a see-saw).

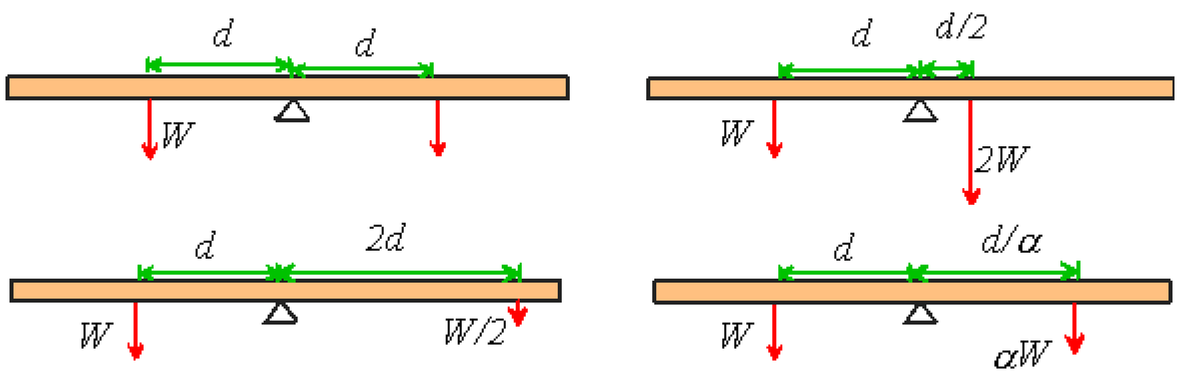


A force applied to a pivoted beam causes the beam to rotate

Hang a weight *W* at some distance *d* to the left of the pivot, and the beam will rotate (counter-clockwise)

To stop the beam rotating, we need to hang a weight on the right side of the pivot. We could

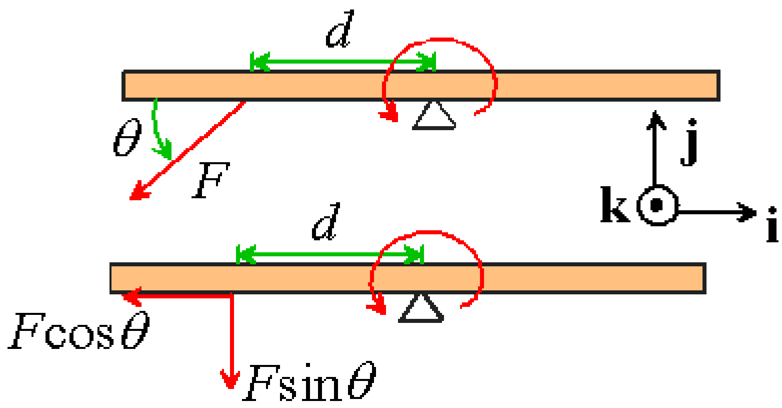
(a) Hang a weight *W* a distance *d* to the right of the pivot
(b) Hang a weight 2*W* a distance *d/2* to the right of the pivot
(c) Hang a weight *W/2* a distance *2d* to the right of the pivot
(d) Hang a weight αW a distance d/α to the right of the pivot.



Four ways to balance the beam

These simple experiments suggest that the turning tendency of a force about some point is equal to the distance from the point multiplied by the force. This is certainly consistent with $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$

To see where the cross product in the definition comes from, we need to do a rather more sophisticated experiment. Let's now apply a force *F* at a distance *d* from the pivot, but now instead of making the force act perpendicular to the pivot, let's make it act at some angle. Does this have a turning tendency *Fd?*



For a force applied at an angle, the turning tendency is $dF\sin\theta$

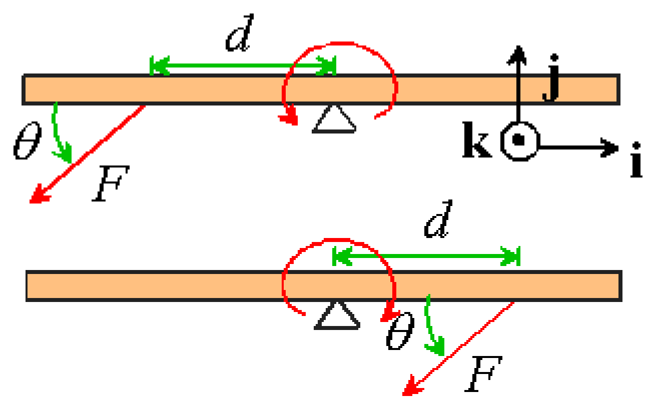
A little reflection shows that this cannot be the case. The force *F* can be split into two components – $F\sin\theta$ perpendicular to the beam, and $F\cos\theta$ parallel to it. But the component parallel to the beam will not tend to turn the beam. The turning tendency is only $dF\sin\theta$.

Let's compare this with $\mathbf{M}_O = \mathbf{r}\times\mathbf{F}$. Take the origin at the pivot, then

$$\mathbf{r} = -d\mathbf{i} \quad \mathbf{F} = -F\cos\theta\mathbf{i} - F\sin\theta\mathbf{j} \quad \Rightarrow \quad \mathbf{r}\times\mathbf{F} = dF\sin\theta\mathbf{k}$$

so the magnitude of the moment correctly gives the magnitude of the turning tendency of the force. That's why the definition of a moment needs a *cross product*.

Finally we need to think about the significance of the *direction* of the moment. We can get some insight by calculating $\mathbf{M}_O = \mathbf{r}\times\mathbf{F}$ for forces acting on our beam to the right and left of the pivot



For the force acting on the left of the pivot, we just found

$$\mathbf{r} = -d\mathbf{i} \quad \mathbf{F} = -F\cos\theta\mathbf{i} - F\sin\theta\mathbf{j} \quad \Rightarrow \quad \mathbf{r}\times\mathbf{F} = dF\sin\theta\mathbf{k}$$

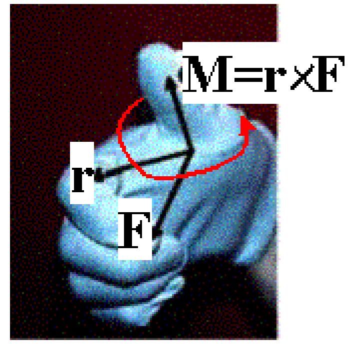
For the force acting on the right of the pivot

$$\mathbf{r} = d\mathbf{i} \quad \mathbf{F} = -F\cos\theta\mathbf{i} - F\sin\theta\mathbf{j} \quad \Rightarrow \quad \mathbf{r}\times\mathbf{F} = -dF\sin\theta\mathbf{k}$$

Thus, the force on the left exerts a moment along the +**k** direction, while the force on the right exerts a moment in the –**k** direction.

Notice also that the force on the left causes counterclockwise rotation; the force on the right causes clockwise rotation. Clearly, the direction of the moment has something to do with the direction of the turning tendency.

Specifically, *the direction of a moment specifies the axis associated with the rotational force, following the right hand screw convention.*



It's best to use the screw rule to visualize the effect of a moment – hold your right hand as shown, with the thumb pointing along the direction of the moment. Your curling fingers (moving from your palm to the finger tips) then indicate the rotational tendency associated with the moment. Try this for the beam problem. With your thumb pointing along +**k** (out of the picture), your fingers curl counterclockwise. With your thumb pointing along –**k**, your fingers curl clockwise.

4.5 Relationships between moments and rotations - A preview of some dynamics

Introductory courses generally try to tell the truth, but do not always tell the whole truth. If you want to know the full truth about moments and rotational motion, read on. If you would prefer to live in a state of blissful ignorance, you can skip this section.

In the preceding section, we interpreted the direction of a moment by saying that it specifies the axis about which the moment tends to rotate a body.

This is a correct interpretation, but the full story of rotational motion is much more complicated. It is very tempting to take the argument a step further, and deduce that something like Newton's law exists for rotational motion of a rigid body. That is to say, just as we can write

$$\mathbf{F} = m\mathbf{a}_c$$

where **F** is the resultant force acting on the body, *m* is its mass, and $\mathbf{a}_c$ is the acceleration of the center of mass, we might guess that

$$\mathbf{M}_C = \Lambda\boldsymbol{\alpha}$$

where $\mathbf{M}_C$ is the resultant moment about the center of mass, $\boldsymbol{\alpha}$ is the angular acceleration (rate of change of angular velocity) and Λ is some measure of the rotational inertia of the solid (equivalent to mass, but for rotations).

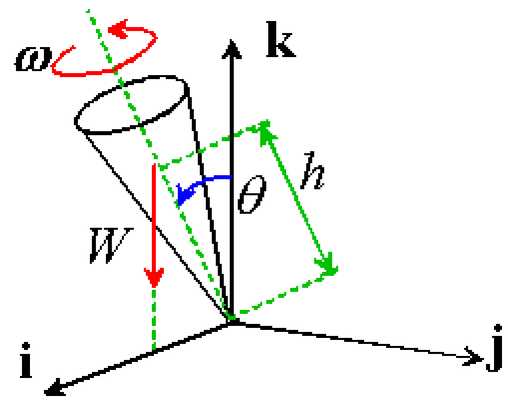
There *is* a law like this, but the correct equation is

$$\mathbf{M}_C = \mathbf{I}\cdot\boldsymbol{\alpha} + \boldsymbol{\omega}\times\mathbf{I}\cdot\boldsymbol{\omega}$$

where **I** is the inertia tensor of the body, $\boldsymbol{\alpha}$ is the angular acceleration of a body and $\boldsymbol{\omega}$ is its angular velocity. (You don't want to know what an inertia tensor is!)

An important consequence of this law is that, *the angular acceleration of a rigid body subjected to a resultant moment* $\mathbf{M}_C$ *will not, in general, be parallel to the direction of* $\mathbf{M}_C$.

The most striking demonstration of this rather counter-intuitive behavior is that a spinning top, or gyroscope, will not fall down.



Thus, the conical spinning top (assumed to be on a frictionless table-top) shown in the picture is subjected to a moment $\mathbf{M}_c = Wh\sin\theta\,\mathbf{j}$. If the top were not spinning, its angular acceleration would be about the **j** axis, and it would fall down. But because it's spinning, its angular acceleration turns out to be about the **k** axis instead, and it *precesses* slowly about the vertical axis.

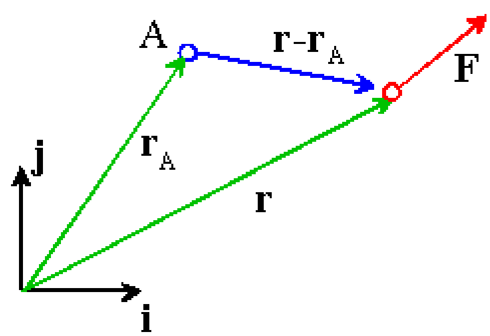
All these subtle issues are way beyond the scope of this course (and even beyond the scope of the freshman dynamics course). For now, you should note that

1. A moment is a rotational force – a `force' that causes rotation.
2. It is correct to interpret the direction of a moment as specifying the axis associated with its rotational driving force;
3. A moment $\mathbf{M}_c$ acting about the center of mass of a rigid body will *sometimes* induce an angular acceleration that is parallel to $\mathbf{M}_c$, but the angular acceleration could have a completely different direction.

4.6 A few tips on calculating moments

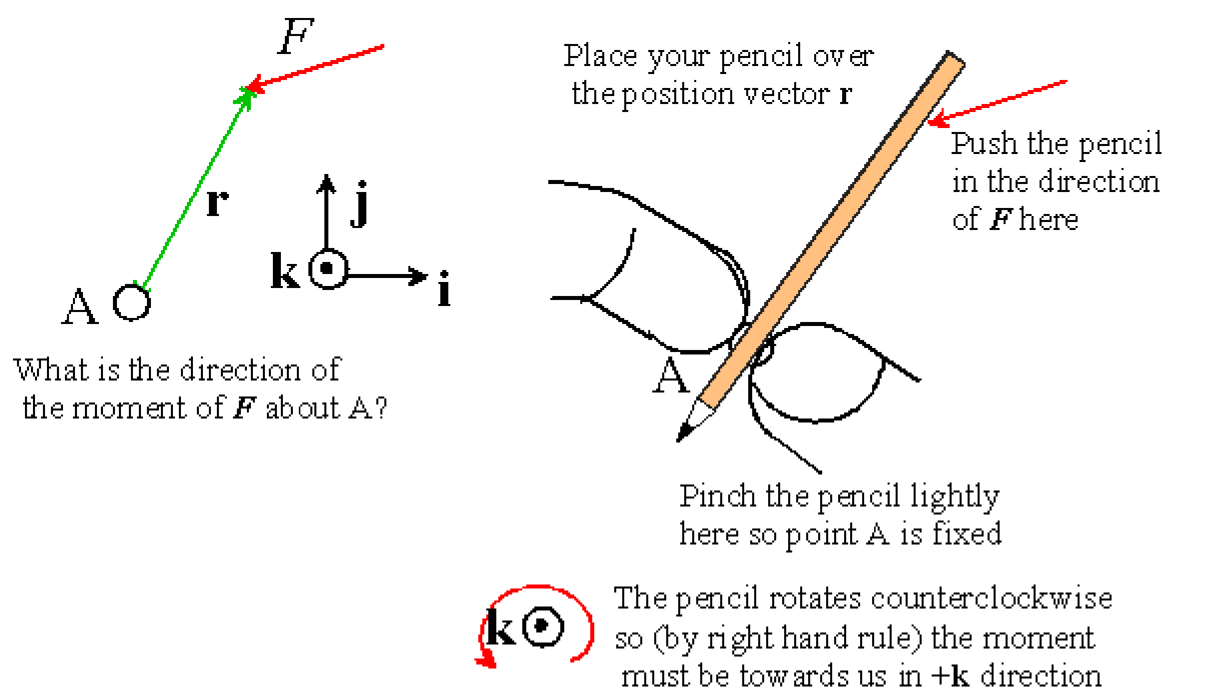
The safest way to calculate the moment of a force is to slog through the $\mathbf{M}_A = (\mathbf{r} - \mathbf{r}_A) \times \mathbf{F}$ formula, as described at the start of this section. As long as you can write down position vectors and force vectors correctly, and can do a cross product, it is totally fool-proof.

But if you have a good physical feel for forces and their effects you might like to make use of the following short cuts.



1. *The direction of a moment is always perpendicular to both* $(\mathbf{r}-\mathbf{r}_A)$ *and* **F.** *For 2D problems,* $(\mathbf{r}-\mathbf{r}_A)$ *and* **F** *lie in the same plane, so the direction of the moment must be perpendicular to this plane.*

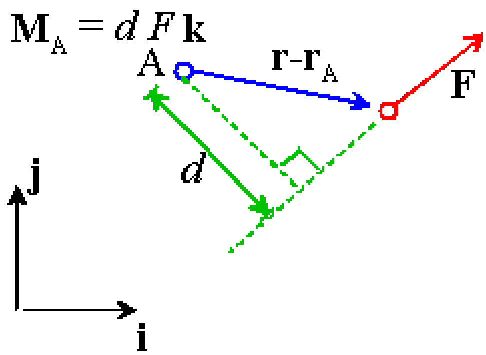
Thus, a set of 2D forces in the {**i,j**} plane can only exert moments in the $\pm\mathbf{k}$ direction – this makes calculating moments in 2D problems rather simple; we just have to figure out whether the sign of a moment is positive or negative.



You can do a quick experiment to see whether the direction is +**k** or –**k.** Suppose you want to find the direction of the moment caused by **F** in the picture above about the point A. To do so,

(i) Place your pencil on the page so that it lies on the line connecting A to the force.
(ii) Pinch the pencil lightly at A so it can rotate about A, but A remains fixed.
(iii) Push on the pencil in the direction of the force at B. If the pencil rotates counterclockwise, the direction of the moment of **F** about A is out of the picture (usually +**k)**. If it rotates clockwise, the direction of the moment is into the picture (–**k).** If it doesn't rotate, you're either holding the pencil in a death grip at A (then the experiment won't work) or else the force must be acting along the pencil – in this case the moment is zero.

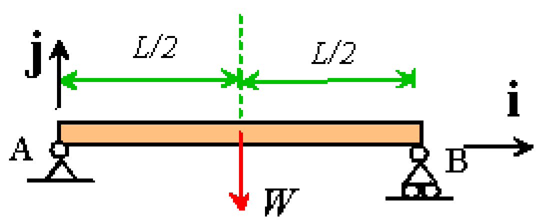
In practice you will soon find that you can very quickly tell the direction of a moment (in 2D, anyway) just by looking at the picture, but the experiment might help until you develop this intuition.



2. *The magnitude of a moment about some point is equal to the perpendicular distance from that point to the line of action of the force, multiplied by the magnitude of the force*.

Again, this trick is most helpful in 2D. Its use is best illustrated by example. Let's work through the simple 2D example problems again, but now use the short-cut.

Example 1: The beam shown below is uniform and has weight *W*. Calculate the moment exerted by the gravitational force about points A and B.



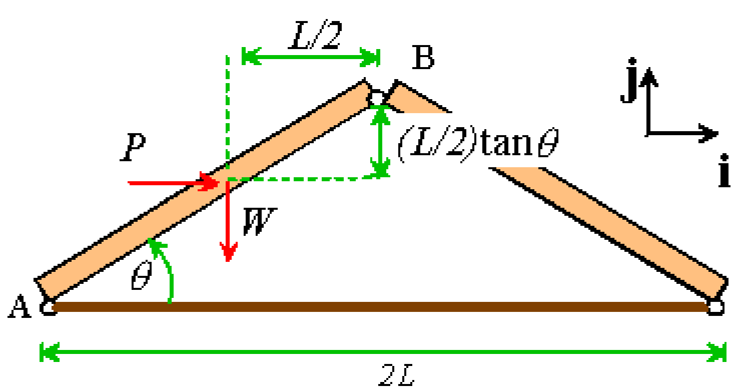
The perpendicular distance from a vertical line through the CG to A is *L/2*. The pencil trick shows that *W* exerts a clockwise moment about A. Therefore

$$\mathbf{M}_A = -(LW/2)\mathbf{k}$$

Similarly, the perpendicular distance to B is *L/2*, and *W* exerts a counterclockwise moment about B. Therefore

$$\mathbf{M}_B = (LW/2)\mathbf{k}$$

Example 2. Member AB of a roof-truss is subjected to a vertical gravitational force *W* and a horizontal wind load *P*. Calculate the moment of the resultant force about B.



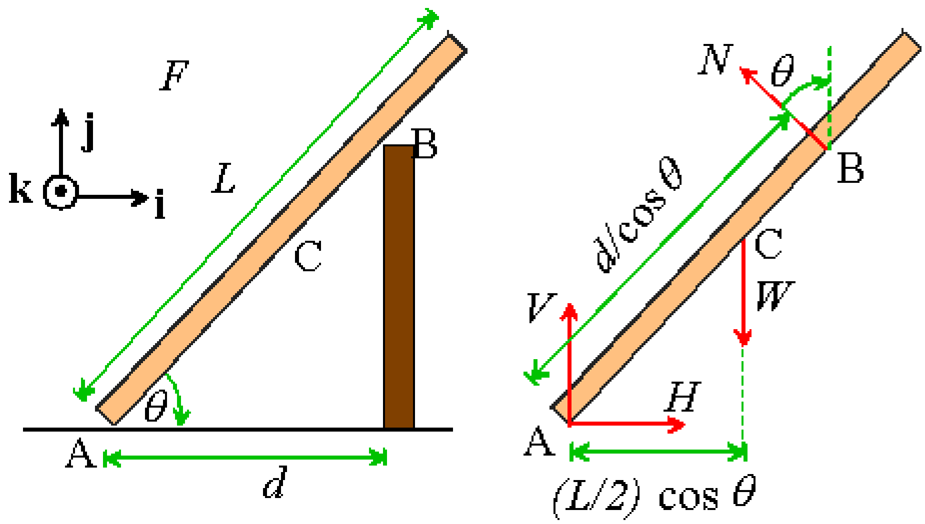
The perpendicular distance from the line of action of *W* to B is *L/2*. *W* exerts a counterclockwise moment about B. Therefore *W* exerts a moment $\mathbf{M}_B = (L/2)W\mathbf{k}$

The perpendicular distance from the line of action of *P* to B is $(L/2)\tan\theta$. *P* also exerts a counterclockwise moment about B. Therefore $\mathbf{M}_B = (L\tan\theta/2)P\mathbf{k}$

The total moment is

$$\mathbf{M}_B = (L/2)\{W + P\tan\theta\}\mathbf{k}$$

Example 4: It is traditional in elementary statics courses to solve lots of problems involving ladders (oh boy! Aren't you glad you signed up for engineering?) . The picture below shows a ladder of length *L* and weight *W* resting on the top of a frictionless wall. Forces acting on the ladder are shown as well. Calculate the moments about point A of the reaction force at B (which acts perpendicular to the ladder) and the weight force at C (which acts at the center of gravity, half-way up the ladder).



The perpendicular distance from point A to the line along which *N* acts is $d/\cos\theta$. The pencil experiment (or inspection) shows that the direction of the moment of *N* about A is in the +**k** direction. Therefore the trick (perpendicular distance times force) gives

$$\mathbf{M}_A = (d/\cos\theta)\ N\ \mathbf{k} \qquad \text{(for the force acting at B)}$$

The perpendicular distance from point A to the line along which *W* is acting is $(L/2)\cos\theta$. The direction of the moment is –**k.** Therefore

$$\mathbf{M}_A = -(L/2)\cos\theta\ W\ \mathbf{k} \qquad \text{(for the weight force)}$$

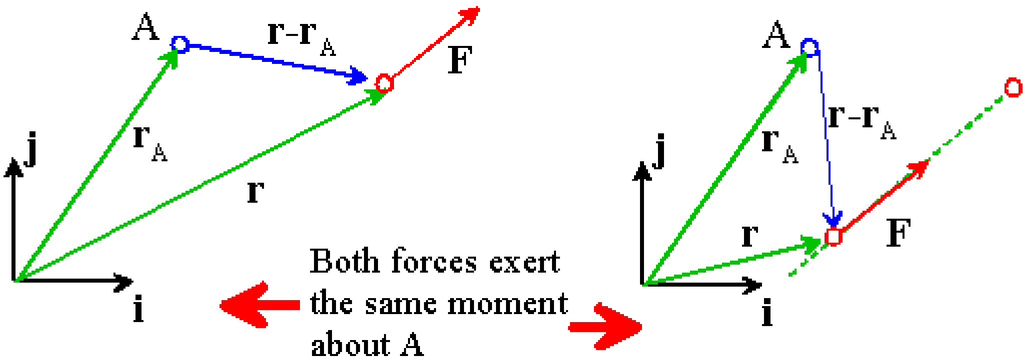
Let's compare these with the answer we get using $\mathbf{M}_A = (\mathbf{r} - \mathbf{r}_A) \times \mathbf{F}$. We can take the origin to be at A to make things simple. Then, for the force at B

$$\begin{aligned}
&(\mathbf{r}_B - \mathbf{r}_A) = d\mathbf{i} + d\tan\theta\mathbf{j} \\
&\mathbf{F}_B = -N\sin\theta\mathbf{i} + N\cos\theta\mathbf{j} \\
&\Rightarrow \mathbf{M} = (\mathbf{r}_B - \mathbf{r}_A)\times\mathbf{F}_B = \left(dN\cos\theta + d\tan\theta N\sin\theta\right)\mathbf{k} \\
&\qquad = \left(dN(\cos^2\theta + \sin^2\theta)/\cos\theta\right)\mathbf{k} = (dN/\cos\theta)\mathbf{k}
\end{aligned}$$

giving the same answer as before, but with a whole lot more effort!

Similarly, for the weight force

$$\begin{aligned}
&(\mathbf{r}_C - \mathbf{r}_A) = (L/2)\cos\theta\mathbf{i} + (L/2)\sin\theta\mathbf{j} \\
&\mathbf{F}_C = -W\mathbf{j} \\
&\Rightarrow \mathbf{M} = (\mathbf{r}_C - \mathbf{r}_A)\times\mathbf{F}_B = -(L/2)\cos\theta\ W\ \mathbf{k}
\end{aligned}$$

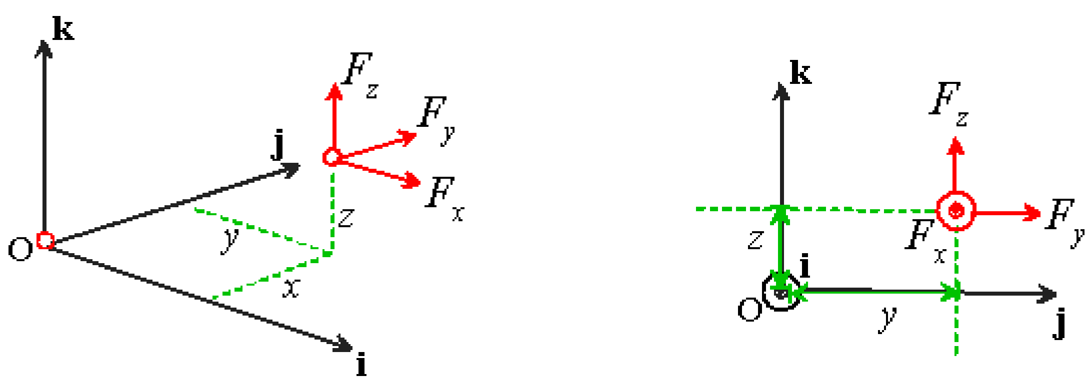


3. *The moment exerted by a force is unchanged if the force is moved in a direction parallel to the direction of the force.*

This is rather obvious in light of trick (2), but it's worth stating anyway.

4. *The component of moment exerted by a force about an axis through a point can be calculated by (i) finding the two force components perpendicular to the axis; then (ii) multiplying each force component by its perpendicular distance from the axis; and (iii) adding the contributions of each force component following the right-hand screw convention.*

The wording of this one probably loses you, so let's start by trying to explain what this means.



First, let's review what we mean by the *component of a moment about some axis*. The formula for the moment of a force about the origin is

$$\mathbf{M}_O = \left\{yF_z - zF_y\right\}\mathbf{i} + \left\{zF_x - xF_z\right\}\mathbf{j} + \left\{xF_y - yF_x\right\}\mathbf{k}$$

This has three components - $M_x = \left\{yF_z - zF_y\right\}$ about the **i** axis, $M_y = \left\{zF_x - xF_z\right\}$ about the **j** axis, and $M_z = \left\{xF_y - yF_x\right\}$ about the **k** axis.

The trick gives you a quick way to calculate *one* of the components. For example, let's try to find the

i component of the moment about the origin exerted by the force shown in the picture.

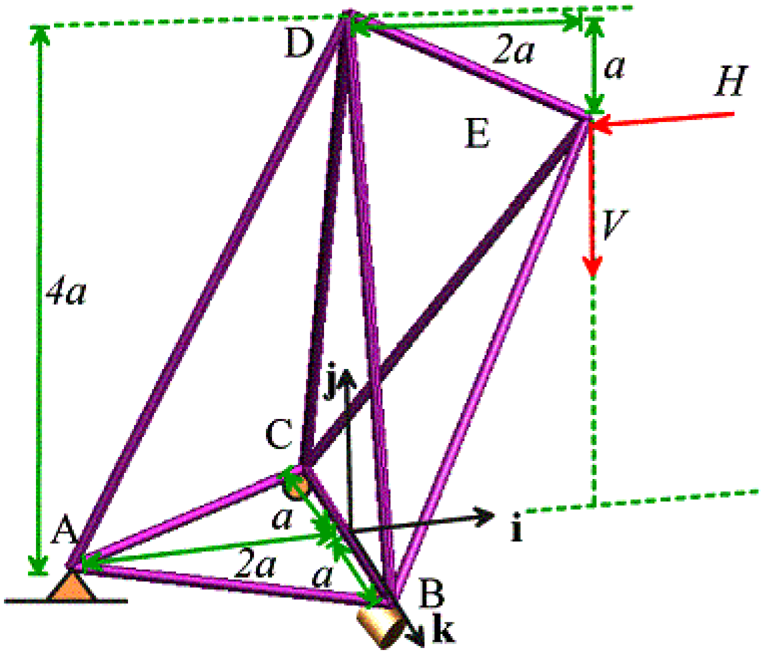
The rule says

(i) Identify the force components perpendicular to the **i** axis – that's F_z and F_y in this case;

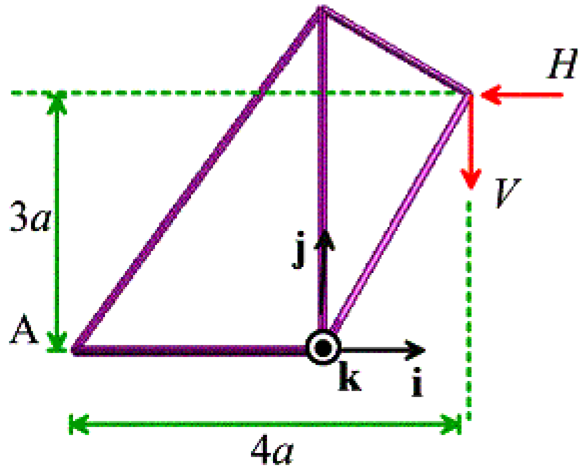
(ii) Multiply each force component by its perpendicular distance from the axis. Drawing a view down the **i** axis is helpful. From the picture, we can see that F_z is a distance *y* from the axis, and F_y is a distance *z* from the axis. The two contributions we need are thus yF_z and zF_y.

(iii) Add the two contributions according to the right hand screw rule. We know that each force component exerts a moment $\pm\mathbf{i}$ - we have to figure out which one is +**i** and which is –**i**. We can do the pencil experiment to figure this out – the answer is that F_z exerts a moment along +**i,** while F_y causes a moment along –**i**. So finally $M_x = \{yF_z - zF_y\}$.

Example: The structure shown is subjected to a vertical force *V* and horizontal force *H* acting at *E*. Calculate the **k** component of moment exerted about point A by the resultant force.



Our trick gives the answer immediately. First, draw a picture looking down the **k** axis



Clearly, the force H exerts a **k** component of moment $3aH\mathbf{k}$, while the force V exerts a **k** component of moment $-4aV\mathbf{k}$. The total **k** component of moment is

$$M_{Az} = a(3H - 4V)$$

This trick clearly can save a great deal of time. But to make use of it, you need excellent 3D visualization skills.